Lenses and Mirrors
PART II
MODIFIED LENS VALUES
Rogers

AUTHOR AND PUBLISHER

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LENSES AND MIRRORS

Part II

Modified Lens Values

BY

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Introduction

This booklet will be found to contain a greater variety of algebraic formulas than the former one, but it is algebra of the simplest kind. The symbols employed to represent values and positions are suggestive of what they represent.

For instance, the letter **n** is the prominent consonant of the word "index," and it is used to represent the index of refraction or resistance of a medium, compared with air. But **x** is the final letter of the word, and it is used to represent the final part of an index, the part following the decimal point, as .52 for an index of 1.52.

D is the natural symbol for dioptric power or value, and in that simplest form it represents the true dioptric value. If there is a secondary dioptric value, or a value that pertains to some other agent than a lens, a subletter or other character is used to distinguish it from the true value, and show its special significance. $\mathbf{D_a}$, $\mathbf{D'}$, $\mathbf{D_m}$, $\mathbf{D_{1m}}$, $\mathbf{D_v}$ and others are illustrations.

For the space which separates two lenses of a couplet, **w** represents the width between them. The characters **F** and **F′** to represent the principal foci are also suggestive of the positions they stand for, for they are not values. The symbols **h** and **h′** to represent the nodal points of a couplet are more arbitrarily chosen, but **c** to represent the position of an optical center is suggestive of that point. To represent the

value of the first lens of a couplet, we use the symbol **a**, and for the second lens **b**. These are more simple than **L** and **L′** and make a simpler appearing formula; while **a′** and **b′** are merely variations of **a** and **b**, due to some special position.

When used in computations, the student must familiarize himself, as far as possible, with the significance of position. He must learn that **ab** means the value of **a** multiplied by the value of **b**; and that **a/b** means the value of **a** divided by the value of **b**. In only one formula do we use the expression $\mathbf{a}^2$. This means **a** multiplied by itself, or taken twice as a factor. In subtracting algebraic quantities, the signs are changed and the quantities are added; while in multiplication and division, like signs give a + answer and unlike signs a — answer.

The greatest difficulty for the student who has never taken algebra will be in the transformations of equations. An equation is but an expression of the equality of two quantities or groups of quantities that are unlike only in expression. The transformations are made for the purpose of getting terms of the same kind together, and uniting and simplifying them. The elements of an unknown value are got into the first member, or part before the sign =, and the known elements are put in the second member.

But this cannot be done except upon principle. Some of the simpler of these principles are:

A term may be transposed from one member to the other by changing its sign. This is merely subtracting it from or adding it to both.

All of the signs in both members may be changed together, for this is merely multiplying both by —1.

Adding or subtracting the same quantity to or from both members, or multiplying or dividing both, by the same quantity does not disturb their equality.

All of this leads to reducing the equation to such simple form that the value you wish to get is revealed to you.

Those students who have had no training in algebra at all may know more about it than they think they do, for it is, aside from a few arbitrary rules, the application of common sense to the purpose of proceeding from the known to the unknown in numerical values. It is much easier to understand than the transpositions of compound lenses.

Those students who find difficulty with any algebraic expression or operation should bear in mind that they are within reach of help. There are school teachers everywhere, and often a grammar school boy can give you the help you need. If not, go to the high school teacher, or to one of the second year high school students. Don't let anything get away from you if you can help it. If you can't get the help, write in.

GEORGE A. ROGERS.

Chicago, Oct. 25, 1918.

I

SURFACE ACTION.

The thinnest of lenses has two surfaces, each doing its part in the whole work of the lens. But, while the sum of the two actions is a constant, or always the same, the action of each surface varies for different distances of the object, as well as for different surface curvatures and index of glass. A study of the surface actions is important, as it enables the student to make close calculations.

The entire dioptric value of a lens is derived from the two factors, excessive density of the glass and combined metric curvature of the surfaces. But, to calculate its action surface by surface, the curvature of the waves of light have to be taken into account, and curvature, at incidence, is dependent upon the distance of the object from the lens. But the density factor is also no longer the same at incidence and emergence, but the ratio of light velocities.

Density Factor.

In glass of an index of 1.50, the density factor of dioptric value is .50 or .5; in glass of 1.52, .52 is that factor; in glass of 1.60, .6 is the factor. But, when light passes from air into glass of 1.5 index, it is slowed up but $.5/1.5 = 1/3$, and this is the surface factor. When it emerges from this same glass it is accelerated in velocity $.5/1 = .5$ or $1/2$, and this is the surface factor at emergence. Therefore, for the same index, the surface factors are different. This principle applies to glass of any index. For an index of 1.52 the factors are, at incidence, $.52/1.52$; at emergence, $.52/1 = .52$.

To indicate how this principle applies to the surface action of a lens, suppose the lens in Figure 1 has +5c on its anterior surface and +7c on its posterior surface; and suppose the object point O is 40″ anterior to the first surface. Incident waves are then +1c, and they impinge upon a surface of +5c. Their combined curvature at the anterior pole of the lens is then the sum of the two, or +6c. This is the curvature factor of the action at this surface.

But the density factor is dependent upon the index of the glass. If that is 1.5, then .5 is the density factor. But the ratio of this factor to the density of

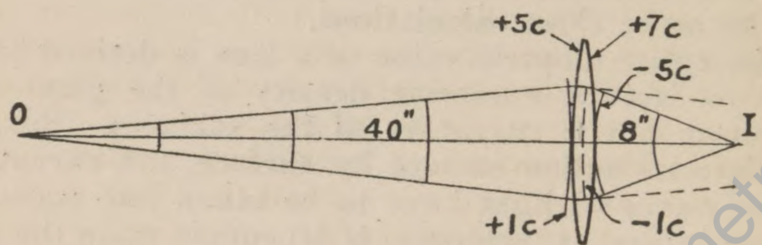

Figure 1.

the glass is .5/1.5 = ⅓. Hence, the action at this surface is ⅓ of +6 = +2 D. The waves going into the glass at this surface are affected in the same manner as they would be by a +2 D. lens. Therefore, —2c is impressed upon them; or they pass into the lens with a curvature of +1 — 2 = —1c.

As the lens is assumed to have no thickness, these —1c waves have still a +7c surface to emerge from. To obtain the curvature factor that is involved at this surface, we deduct or subtract —1c from +7c, or we change the sign of the wave curvature to +1 and add it to +7, making a total of +8c. Here we have the same density factor, .5, but its ratio to the medium into which the waves are to pass is .5/1 = .5. Hence the action at this surface is .5 of +8 = +4 D.

The —1c waves of light in the lens must pass out of it through a surface whose action is +4 D. It will impress —4c upon the emergent waves, or cause them to emerge as —5c waves. Hence, the incident +1c waves are first converted into —1c waves at the first surface, and then to —5c at the second surface. In other words, the action of the front surface is +2 D. and of the back surface is +4 D. making a total action and value of +6 D., and divided between the surfaces as shown. But incident waves are +1c and emergent waves are —5c, which is a change of 6c in their curvature; and this is equal to the combined curvature of both surfaces, +12c, multiplied by .5.

For any other index than 1.50, or for any other surface curvatures or distance of the object, the method of calculating surface action is the same. It may be stated as a rule as follows:

For incident surface, to get dioptric action:

Add incident metric curvature of waves of light from object to metric curvature of incident surface of lens.

Multiply combined metric curvature thus obtained by ratio of excessive density of glass to its index, for action.

Deduct action obtained from incident metric curvature of the waves for curvature of waves in lens.

For emergent surface, to get dioptric action:

Subtract metric curvature of waves in the lens from metric curvature of emergent surface of lens.

Multiply combined metric curvature thus obtained by ratio of excessive density of glass to unity, for action.

Deduct action obtained from metric curvature of waves in the lens before emergence, for final metric curvature.

Second Example.

In Figure 2 the anterior surface of the lens is +10c and its posterior surface is —5c, making the combined metric curvature of its two surfaces +5c. If the index of the glass is 1.6, then the dioptric value of the lens is .6 of +5 = +3 D. If the object is at a distance of 20″, the lens has sufficient power to overcome the divergence of incident rays (+2 D.) and a remaining power (+1 D.) to converge the rays to a focus 40″ posterior to the lens.

In this action the anterior surface acts for +4.50 D.

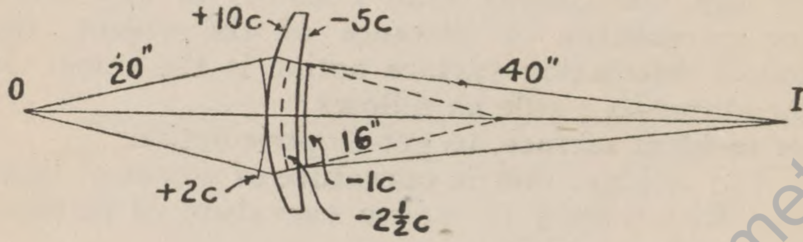

Figure 2.

and the posterior surface for —1.50 D. making the complete action +3 D. The action at the respective surfaces are calculated as follows:

Ant. surface,

$$\begin{array}{l} +10 \\ \underline{+\ 2} \\ +12 \times .6/1.6 = +4.50 \text{ D.} \end{array}$$

Wave frontage in glass is therefore +2 —4.5 = —2.5c.

Post. surface,

$$\begin{array}{l} -\ 5 \\ \underline{-(-2.5)} \\ -2.5 \times .6 = -1.50 \text{ D.} \end{array}$$

This action impresses +1.5c on —2.5c waves in the glass, so that they emerge as —1c waves, and focus at 40″.

The action at the respective surfaces is therefore +4.50 D. at the anterior and —1.50 D. at the posterior surface.

Primary Action.

In each of the examples given, the object is located at a finite distance from the lens. As this distance is varied, the action of the surfaces will vary. We are more accustomed to consider the action of a lens on standard light, or light from infinity, consisting of plane waves or parallel rays.

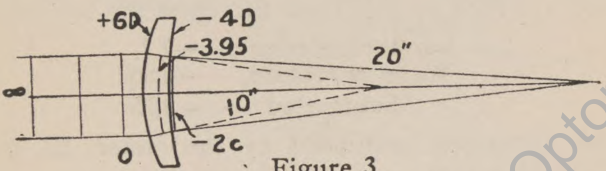

Figure 3.

We have also employed glass of an uncommon index, as the standard crown glass of which most optometric lenses are made has an index of 1.5232, with surfacing tools adapted to that index, or approximately 1.52. Again, the metric curvature of the surfaces is stated, where with regular lenses their values in diopters may be found with a lens measure. It is then an easy matter to determine their metric curvature by a simple calculation.

Taking, then, an ordinary crown glass lens, if a surface measures +5 D. its metric curvature is approximately +5/.52. That is, it is the dioptric value divided by the excess density of the glass. If a +2 D. lens of this kind is made up meniscus on a +6 base

curve, as shown in Figure 3, its posterior surface will measure —4 D. Then for incident parallel rays, the action of the anterior surface is determined as follows:

$$\frac{+6}{.52} \times \frac{.52}{1.52} = \frac{+6}{1.52} = +3.95 \text{ D.}$$

That is, the action of the anterior surface is its value divided by the index of the glass.

But, as the entire value of the lens is +2 D., and the action at the anterior surface exceeds that value 1.95 D., the action at the posterior surface must be such as to neutralize the excess, or —1.95 D. Calculated as heretofore, the curvature of the posterior surface is —4/.52 = —7.7, and therefore the action is

Surface curvature,	—7.7
Wave curvature,	—3.95
Subtracted =	—3.75

This curvature, multiplied by the factor .52, gives —1.95 D. action.

It is only at the incident surface that this complexity, with respect to the index, arises. Whatever the incident surface may do, the action of the posterior surface completes it and makes it equal to the power of the lens. An incident surface will only do its exact value when the object is located at the anterior principal focus of that surface, and then the posterior surface will also do its exact value upon emergent light.

Illustration.

Suppose a lens of any index has +5 D. anterior and —4 D. posterior surfaces, as shown in Figure 4. Then, if the object is at the principal focus of the

anterior surface, 8″ forward of the lens, the anterior surface will parallel the rays in the lens. Then the —4 D. posterior surface will focalize these parallel rays at its own principal focus, 10″ anterior to the lens.

Whatever the index of the glass of which a lens is made, the surface actions for the above position of the object would not be altered. If the index were higher, the metric curvature of the surfaces would be correspondingly less, so that whatever the index of the

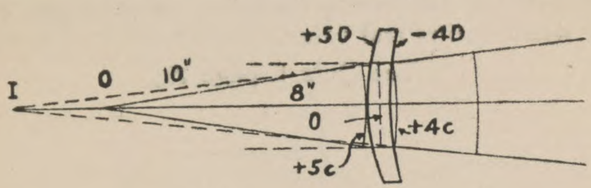

Figure 4.

glass of which a lens is made, an object at the anterior focus of the anterior surface would cause both surfaces to act for their exact value.

Algebraic Formulas.

If **D** represents the dioptric value of a plano-convex lens, with its spherical surface anterior, then **D** is also the dioptric value of the surface. Then,

$$\frac{\mathbf{D}}{\mathbf{x}} = \text{metric curvature of the surface.}$$

$$\frac{\mathbf{D}}{\mathbf{x}} \times \frac{\mathbf{x}}{\mathbf{n}} = \frac{\mathbf{D}}{\mathbf{n}} = \text{action on parallel rays.}$$

But, since the entire action of the lens is **D,** then,

$$\mathbf{D} - \frac{\mathbf{D}}{\mathbf{n}} = \frac{\mathbf{Dn} - \mathbf{D}}{\mathbf{n}} = \frac{\mathbf{D(n-1)}}{\mathbf{n}} = \frac{\mathbf{Dx}}{\mathbf{n}} = \text{action}$$

at plane surface.

The action at the posterior plane surface is therefore **x** times the action at the anterior curved surface; or the two actions are in the ratio of 1 to **x.** This provides the basis for taking the thickness of lenses into account, and calculating the effect of thickness upon the whole action and value.

(See Quiz I, page 79)

II

LENS MIRROR.

If one of the surfaces of a spherical lens is silvered, or coated with the amalgam used in making mirrors, it becomes a lens-mirror. That is, it is both a lens and a mirror. Its dioptric value as a lens-mirror is dependent upon the value of each element that enters into its structure, and these, aside from index, depend upon the curvature given to the surfaces.

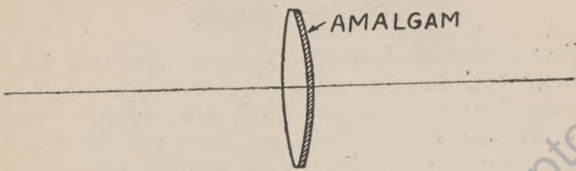

Figure 5.

The optometrist who has a small surfacing plant may make up such a combination of any power he desires, but must observe closely what factors contribute to its dioptric power. Otherwise he may get something he doesn't want, or fail to get what he desires. He is confined, in surfacing the lens, to the standard tools or laps, which are made for glass of an index of 1.52.

The surface to be silvered has a lens value. If convex that value is positive or plus, if concave it is negative or minus. But the metallic coating that is put upon it takes the opposite curvature, and this metallic surface is the mirror. If the lens value is convex or positive, the mirror is concave. But since a

concave mirror is positive in value, the lens value of the surface and its mirror value are both positive. A concave lens surface, if silvered, also gives a negative value to both the lens and the mirror.

But, while the lens value of a silvered surface is only a fractional part of its metric curvature, about ½ or a little more, its mirror value is always just double its metric curvature. As the ratio of the two factors are as ½ to 2 = 4, the mirror value of a surface is practically four times its lens value. But, while light is reflected but the one time from the mirror, it passes twice through the lens, and the lens also has two surfaces, only one of which is silvered.

Simpler Forms.

There are four forms of the lens mirror to be considered. The simpler forms are those in which both or either lens and mirror values are neutral. That is (1) both lens and mirror are neutral; (2) the lens value is neutral; (3) the mirror value is neutral. The more complex form (4) is that in which both lens value and mirror value enter into the power of the lens-mirror.

A plane mirror, made up in the usual manner, is really a lens mirror, but both elements, lens and mirror, are neutral, and its power is 0. If, however, the lens is a meniscus neutral, silvering either surface will give that surface a mirror value of double its metric curvature. To get such value it is only necessary to reduce its dioptric to metric curvature, and double the amount; or to divide double its dioptric value by the excessive density of the glass.

If the dioptric value of the lens surface is represented by **D,** and its mirror value by $\mathbf{D_m}$, then

$$\mathbf{D_m = 2\,D/x.}$$

In other words, divide the lens value of the surface to be silvered by the density of the glass in excess of air (index less 1) to obtain the metric curvature of the surface. Multiplying this metric curvature by 2 gives the mirror value of the surface. If the silvered surface is convex, the mirror is concave and positive; if concave the mirror value is convex or negative.

If the plane surface of a plano-convex or plano-concave lens is silvered, the mirror value of that surface will be neutral or 0. The lens-mirror gets its entire dioptric value from the lens. But, as light must

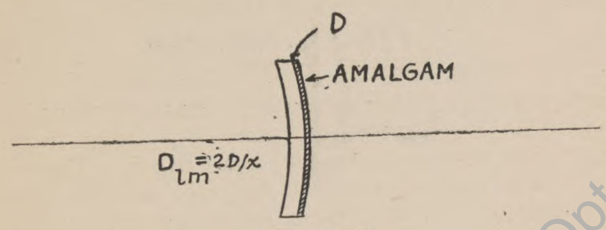

Figure 6.

pass through the lens to get to the mirror, and after reflection it passes back through the lens, it passes twice through the lens. This gives the lens-mirror double the value of the lens alone. A +2 D. plano-convex lens, silvered on its plane surface, has a dioptric value of +4 D. It is as though two +2 D. lenses, with their plane surfaces together, had acted upon the light.

The lens-mirror just described, +4 D., is in reality a mirror. It has but one principal focus, and this is ¼ meter = 10″ anterior to it. As a mirror, all the principles of mirror action, including its production of real or virtual images, apply to it. The object, in order to make it produce a real image, must be at a

greater distance from the mirror than its focal length. The measurement of its action by the focal-length system applies to it the same as to any concave mirror with a 10″ focal-length.

This is really the simplest way of making a spherical mirror of any given power. The index of the glass is not involved, for the dioptric tool that grinds the surface is shaped to the index and gives a definite lens value to the surface. Double this value for the passage of light twice through the lens gives the di-

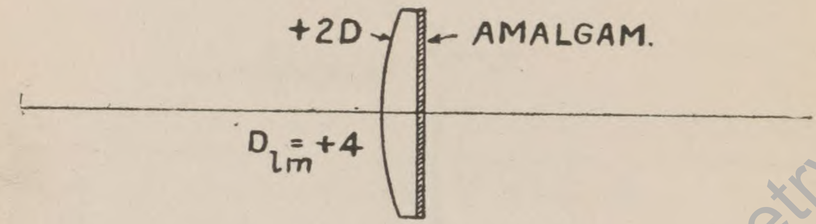

Figure 7.

optric value of the lens-mirror, for the mirror contributes nothing to its power.

Fourth Form.

In the fourth form both lens and mirror are elements in the value of the lens-mirror. The simplest form of this variety is obtained when the curved surface of a plano-spherical lens is silvered. For instance, if the plano-convex lens of +2 D. last described is silvered on the convex surface, that surface is given a mirror value equal to double its metric curvature. To this mirror value is added double the lens power, for light passes twice through the lens.

Its lens value is therefore the same as if the plane surface were silvered. To this is added its mirror value, and that is its metric curvature multiplied by 2. Its metric curvature, for glass of an index of 1.52, is 2/.52, and double this is 4/.52 = +7.69+D. Adding to this value the lens value of +4 D. gives a total lens-mirror value of +11.69 D. This is a considerable dioptric value for so simple a construction. But, this value may be increased by giving the other surface of the lens also a +2 D. curve, which will add +4 D. again to the lens-mirror value.

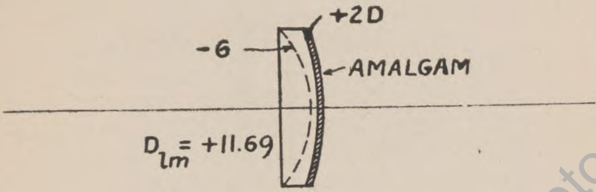

Figure 8.

As a comparatively slight dioptric curvature, if silvered, gives a high lens-mirror power, and both values at the silvered surface are of the same kind, positive or negative, the silvered surface is the most potential one in gaining high power out of a weak curve. But the opposite surface of the lens, since it exercises double its nominal power, is not to be ignored or neglected.

We may, by giving the unsilvered surface a negative curvature and power, counteract or neutralize a part or all of the lens-mirror value of the silvered surface. For instance, to neutralize the +11.69 D. of the above described lens-mirror surface, we have only to grind a minus curve of half of the above value on the

front surface. A —6 D. curve on that surface would convert the entire value into +11.69 — 12.00 = —0.31 D. lens-mirror.

There would be no object in making a construction of the kind, as the purpose of a lens-mirror is to get as high a power of a particular kind as possible with the least complexity of structure, rather than to build a "freak" optic agent. The construction is described for the purpose of making the principle of construction clear. Usually a lens-mirror would be built so as to make all elements of one kind, and all positives.

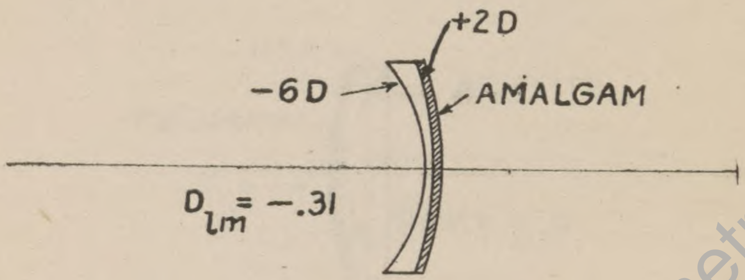

Figure 9.

Unusual Forms.

The unusual forms that still serve a purpose are negative lens-mirrors, or mirrors that distort the images of objects. If a meniscus neutral is silvered on the concave surface, it makes a convex mirror of practically four times the dioptric value of the lens surface that is silvered. That is, a —2 D. surface of 1.52 glass has a mirror value of —4./52 = — 7.69 D. A small hand mirror of this kind gives one a reduced image of himself in the glass.

Distorted images are produced by any irregularity in the surfacing of a lens-mirror, but if the irregular surface is silvered, greater distortion results from a

given irregularity. Cylindrical mirrors, though in a certain sense regular in form, give images that are distorted in dimensions. If the cylinder is plano-convex, and silvered on the plane surface, along its axis it is a plano mirror; but across its axis it has double the power of the cylindrical lens.

If the cylindrical surface is silvered, it is still neutral along the axis of the cylinder; but across the axis it has a power practically six times that of the cylindrical lens. With such a mirror, if held with the axis vertical, one appears as in a plane mirror vertically, but horizontally he is of exaggerated dimensions. He must be within a focal-length of the power to get these results, however. Otherwise he will be erect vertically but inverted horizontally.

The most unique of these effects is obtained by silvering a concave or toric surface, with a convex spherical forward surface of the same power as the axis. This gives one a reduced image of himself in the mirror, all dimensions reduced, but that meridian at right angles to the axis more reduced than the other. Consequently, by shifting the position of the axis, one may make himself appear short and fat or lank and thin. At an oblique position he of course appears lop-sided, or contorted and mis-shaped.

The distance of the observer from the mirror has to be taken into account to determine results with accuracy. But there can be no doubt of results when one tests the agent practically, although they may not be what he expects.

Algebraic Formulas.

In a lens-mirror there are two lens surfaces whose combined dioptric lens value we represent by **D.** But, as only one of these surfaces is silvered, we represent the dioptric lens value of the forward and unsilvered

surface by **a,** and the dioptric lens value of the silvered surface by **b.** $\mathbf{D}_m$ continues to be the symbol for the mirror value only of the silvered surface.

Then, as heretofore shown,

$$\mathbf{D}_m = 2\ \mathbf{b}/\mathbf{x}.$$

For, $\mathbf{b}$ = the lens value of silvered surface,
and $\mathbf{b}/\mathbf{x}$ = metric curvature of silvered surface,
$\therefore 2\mathbf{b}/\mathbf{x}$ = mirror value of silvered surface.

But the lens value of the silvered surface is **2b,** and hence the total lens and mirror value of that surface is the sum of the two, but

$$2\mathbf{b} + 2\mathbf{b}/\mathbf{x} = \frac{2\mathbf{bx} + 2\mathbf{b}}{\mathbf{x}} = \frac{2\mathbf{b}(\mathbf{x}+1)}{\mathbf{x}} = \frac{2\mathbf{bn}}{\mathbf{x}}$$

This is the combined lens and mirror value of the silvered surface. But to this there is still to be added **2a** to complete the full value of the lens-mirror.

If we represent the entire dioptric value of the lens-mirror by $\mathbf{D}_{lm}$, then we will have as a complete formula for its value,

$$\mathbf{D}_{lm} = 2\mathbf{a} + 2\mathbf{bn}/\mathbf{x}$$

Factoring and reducing the above we have

$$\mathbf{D}_{lm} = \frac{2\mathbf{ax} + 2\mathbf{bn}}{\mathbf{x}}, \text{ or}$$

$$\mathbf{D}_{lm} = \frac{2(\mathbf{ax} + \mathbf{bn})}{\mathbf{x}}$$

But this reduces to

$$\mathbf{D}_{lm} = 2\mathbf{D} + 2\mathbf{b}/\mathbf{x}$$

Hence, the total lens and mirror value of a lens-mirror is twice the full lens value of the lens, plus twice the lens value of the silvered surface, divided by the excess density of the glass of which the lens is made. Its focal-length is of course the reciprocal of its full dioptric value.

(See Quiz II, page 80)

III

SUBMERGED LENS.

The nominal dioptric value of a lens of a given form is its value when acting in or surrounded by air, the standard **base** for the action of a lens. If the glass in the lens has an index of 1.5, its resistance to the

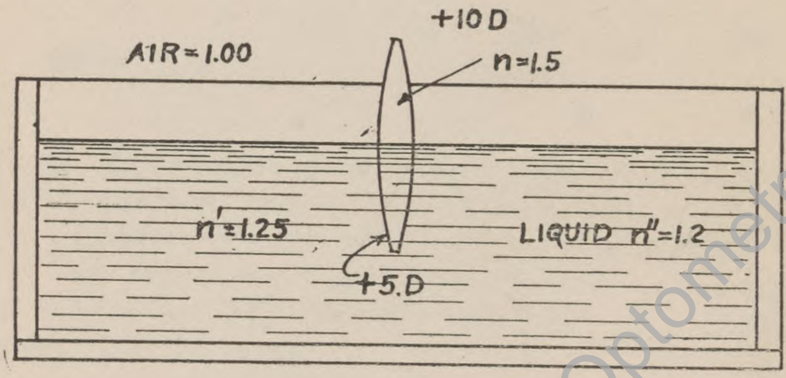

Figure 10.

propagation of light in it is 1.5 times as great as the resistance of air, or .5 greater. It is this excess resistance in comparison with the resistance of the **base** that establishes its value.

As air is the standard of resistance, its index is unity or 1.00. The ratio of .5 to 1, or $.5/1 = .5$. But the same lens, if submerged in a medium whose index is 1.2, has but ½ its nominal value. This is due to the fact that the **base** is no longer air, but a medium with .2 greater resistance than air. The

index of the glass in this medium is 1.5/1.2 = 1.25, so that the excess resistance of the glass, over this **base,** is .25. The same result is obtained by getting the difference between .5 and .2 and obtaining its ratio to the **base,** as .3/1.2 = .25.

A +10 D. lens of 1.5 glass, if submerged in a medium whose index is 1.20, therefore has a dioptric value of +5 D. in that medium. Observe that while .5 is the excess resistance of the glass, compared with air; and .2 is the excess resistance of the fluid, compared with air; .25 is the excess resistance of the glass compared with the fluid, and it is the density factor of dioptric value. This factor is not the difference of indexes merely, but the ratio of that difference to the **base.**

When a glass lens is completely surrounded by some other fluid than air, so that incident and emergent light comes from and passes into that medium, and object and image are situated in it, it is **completely submerged.** If the less dense medium is surrounded by the more dense, as a water bubble in glass, this is **counter submergence.** If the denser medium is merely surrounded by a shell if lighter density, so that light passes from it into air again, it is merely an **enclosed lens.** If a lens has air in one direction from it, but water or other medium denser than air in the other direction, it is **semi-submerged.**

Each of these different phases or degrees of submergence gives a different result and value to a lens of a fixed form and index. It is not the lens that has changed, except in situation, but the **base.** The lens may also be of different forms, and its index relative to air may be varied in different cases. As we have already made some progress in under-water observation, in which lenses play a part, and may wish to extend this field, the principles involved should be well understood by those in optical lines.

1. Complete Submergence.

To obtain the dioptric value of a glass lens that is completely submerged, if we let **D** represent its dioptric value in air, and **D′** its dioptric value in the submerging medium; **n** and **x** the index and excess of the glass in an air **base; n′** and **x′** the relative index and excess; and **n″** and **x″** the index and excess of the real **base** medium, relative to air, the dioptric value of the glass lens in this medium is obtained by either Formula 7 or 8:

1. $\mathbf{n} = \mathbf{n}'\mathbf{n}''$
2. $\mathbf{n}' = \mathbf{n}/\mathbf{n}''$
3. $\mathbf{n}'' = \mathbf{n}/\mathbf{n}'$
4. $\mathbf{x} = \mathbf{n} - 1$
5. $\mathbf{x}' = \mathbf{n}' - 1$
6. $\mathbf{x}'' = \mathbf{n}'' - 1$

7. $$\mathbf{D}' = \frac{\mathbf{D}}{\mathbf{x}}\left(\frac{\mathbf{n}}{\mathbf{n}''} - 1\right)$$

8. $$\mathbf{D}' = \frac{\mathbf{D}}{\mathbf{x}}\left(\frac{\mathbf{x} - \mathbf{x}''}{\mathbf{n}''}\right)$$

According to the 7th formula, if a lens of glass whose index is 1.5 is submerged in a fluid whose index is 1.25, the relative index is $\mathbf{n}/\mathbf{n}'' = 1.5/1.25 = 1.2$. Deducting 1 from this index gives .2 as the density factor. But, according to the 8th formula $\mathbf{x} - \mathbf{x}'' = .5 - .25 = .25$, and this difference divided by **n″** gives $.25/1.25 = .2$, the density factor as before.

In air the density factor is .5, in the submerged lens it is .2, or ⅖ as great. Hence the lens, whatever its power in air, has ⅖ or .4 of that power in

the fluid. If the original lens was +10 D., then the submerged lens is .4 of $+10 = +4$ D. Submerged it has the same ratio to its air value as these excessive density factors. Expressed in a proportion.

$$\mathbf{D} : \mathbf{D'} :: \mathbf{x} : \mathbf{x'}$$

In the same kind of construction, the submerged value of the lens may be given, and its air value be called for; or the air value and its submerged value may be given, and the index of the fluid be called for. We will analyze two such problems to show how they may be solved, using the same known factors as above.

1. If the dioptric value of a lens, submerged in a fluid whose index is 1.25, is +4 D., what is its dioptric value in air, if the index of the glass in air is 1.5? Here we have **x** and **x″**, and **D′** and **n** and **n″**. What we are to find is **D**. By either formula we may determine **n′** and **x′**. According to the first formula, $\mathbf{n'} = 1.2$, and $\mathbf{x'} = .2$. But .2 is .4 of .5. Therefore +4 is .4 of **D**. Hence $\mathbf{D} = +4/.4 = +10$ D.

2. If a lens of 1.5 glass is +10 D. in air; but submerged in a certain fluid its dioptric value in the fluid is +4 D., what is the index of the fluid? Since the reduction in value by submersion leaves it but .4 of its value in air, the density factor in the fluid is .4 of $.5 = .2$, and the relative index is therefore 1.2. The index of the fluid is therefore $1.5/1.2 = 1.25$.

3. If a lens has +10 D. power in air; but submerged in a fluid of 1.25 index its power is +4 D., what is the index of the glass in the lens? From the ratio of values, the density factor in air is $10/4 = 2.5$ of that factor in the fluid. But, in the fluid, it is the excess of the relative index that provides the factor, and we do not know that, nor have we the other two factors that provide it.

But, using the second formula, substituting numerical values where we have them, and using the

symbols where they are lacking, we obtain the following equation:

$$\frac{10}{\mathbf{x}}\left(\frac{\mathbf{x} - .25}{1.25}\right) = +4.$$

Simplifying the first member by performing the multiplication indicated, gives us,

$$\frac{10\mathbf{x} - 2.5}{1.25\mathbf{x}} = +4.$$

Clearing the last equation of fractions, we have

$$10\mathbf{x} - 2.5 = 5\mathbf{x}.$$

From the last equation, by transpositions and reductions, we obtain the following:

$$5\mathbf{x} = 2.5, \text{ and}$$
$$\mathbf{x} = .5, \text{ whence}$$
$$\mathbf{n} \doteq 1.5.$$

2. Counter Submergence.

In this class the medium of lesser density is surrounded by the medium of greater density, so that the latter is the **base** medium. The effect of this is to give the lens form that is counter-submerged an index less than unity. For instance, an air bubble in the form of a lens, imprisoned in glass of an index of 1.5, has an index, relative to the glass that surrounds it, of $1/1.5 = .66\frac{2}{3}$. The density factor is therefore $.66\frac{2}{3} - 1 = -.33\frac{1}{3}$. The other factor is the metric curvature of the air lens.

If a water lens is similarly imprisoned in glass of an index of 1.5, the relative index of the water lens is $1.33\frac{1}{3}$ divided by $1.5 = .88^{8}/_{9}$; and this index less unity is $-.11\frac{1}{9}$, the density factor. By the second formula, $\mathbf{x} = .33\frac{1}{3}$, $\mathbf{x}' = .5$ Hence, $\mathbf{x} - \mathbf{x}' = -.16\frac{2}{3}$, and this divided by $\mathbf{n}$, or $1.33\frac{1}{3}$, is $-.11\frac{1}{9}$, or the same

result as obtained by the first formula. The other factor is metric curvature. Observe that the index of submersion and counter submersion are reciprocals, each of the other as always in reciprocals.

3. Enclosed Lens.

If a glass lens is merely enclosed by a medium of less density than itself, so that the light passes from

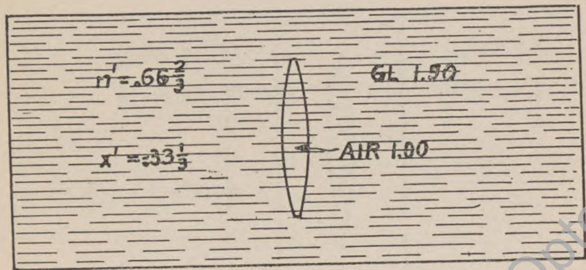

Figure 11.

air into the enclosed lens, and emerges again into air, air again becomes the **base,** and the index for the glass lens so enclosed is merely the difference between the index of the glass and that of the enclosing medium, for the divisor 1 will not affect the result.

If a glass lens of 1.5 index is enclosed by a jelly of the index of water, the density factor of value is $1.5 - 1.33\frac{1}{3} = .16\frac{2}{3}$. As this factor is $\frac{1}{3}$ of .5 the lens so enclosed will have a dioptric value $\frac{1}{3}$ of its dioptric value in air, or a +12 D. lens will have a value of +4 D. But the surfaces of the enclosing jelly must be plane on the outside. Otherwise there will be a jelly lens enclosing one made of glass, and

its value will have to be added to that of the enclosed glass lens. The jelly lens is computed as a lens in air.

The Kryptok bifocal is an example of this sort of combination of media. In the "fused" bifocals one surface of the segment is within the less dense glass and the other surface is exposed to the air. But, as the latter exposed surface has the same curvature as the surface of the outer lens, it increases the power of the surface over the exposed area the same as the

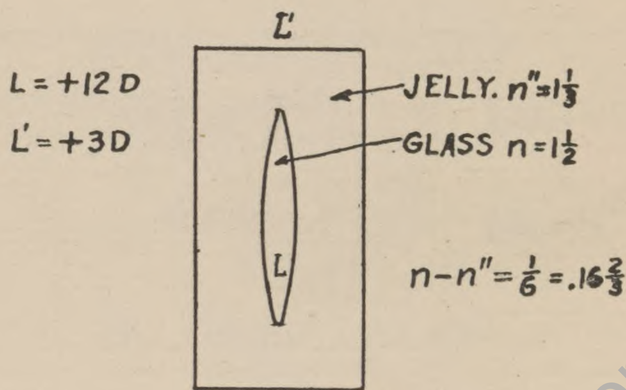

Figure 12.

inner surface, so that the increase in value due to the higher index has the same density factor of increase as the surface concealed within the main lens.

If the main lens has an index of 1.5232, and the enclosed lens an index of the segment is 1.6104, then $\mathbf{x} = .6104$ and $\mathbf{x}'' = .5232$, and $\mathbf{x} - \mathbf{x}'' = .0872$. We do not divide this difference by $\mathbf{n}''$, according to the second formula, for the emergent medium is air, index 1, and 1 as a divisor leaves the density factor unchanged. As the factor is .0872 is ⅙ of the factor .5232, it will take 6 D. of the standard glass to make 1 D. addition of the special glass. Hence the ratio of curvatures for diopters added is 1 to 6.

4. Semi-Submergence.

If air is the incident medium for a glass lens, and some liquid is the emergent medium, the surface of emergence is submerged while the incident surface is not. Hence, the lens is semi-submerged. Taking glass of 1.5 for the lens, and a liquid of 1.2 for the emergent medium, it is evident that the lens will be given a dual value, one in relation to air and the other in relation to the liquid.

A +12 D. glass lens, of this glass, has +24c. But,

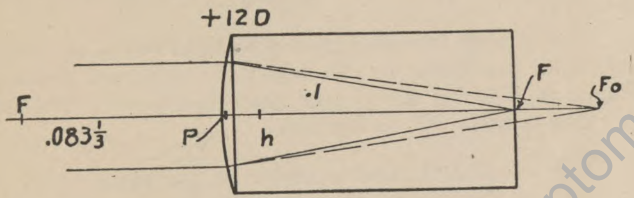

Figure 13.

to calculate its dual values we shall have to know which surface is submerged and what curvature each surface has, for the submerged surface will have less value for equal curvatures than the one exposed to air. If our lens is a plano convex, with its convex surface toward the air, the plane surface can have no dioptric value. But what is the value of the curved surface in this situation, or of the lens?

Incident parallel rays of light will be modified at incidence ⅓ of +24 = +8D. If the posterior plane surface allowed the light to pass out into air, ½ of +8 or +4 would be added to this value, making +12 D. in all. The surface is plane, but the emergent

medium is not air. Its index, relative to air, is 1.2, but the index of the glass relative to this medium is $1.5/1.2 = 1.25$. At the emergent plane surface .25 will be added to the action of the anterior surface, and .25 of $+8 = +2$ D. The complete action of both surfaces is therefore $+8$ increased by $+2 = +10$ D.

Emergent parallel rays from the liquid will not be modified at the plane surface, as there is no metric curvature in surface or light waves. But, at the anterior convex surface, the full action of $+12$ D. will take place. Hence, the dual values of the lens in this situation are $+10$ and $+12$ D. Their focal-lengths will correspond, or be the reciprocals of the dioptric powers.

This places the posterior principal focus at a point 100 mm. posterior to the lens; the anterior principal focus at a point 83⅓ mm. anterior to it. We may lay the system off as follows:

D $= +12$ D., the anterior or air value.
f $= 83\frac{1}{3}$ mm., the anterior focal-length.

D′ $= +10$ D., the posterior or liquid value.
f′ $= 100$ mm., the posterior focal-length.

F = the anterior principal focus.
F′ = the posterior principal focus.

In this system, there is a point on the principal axis at a distance of **f** from **F′**, and **f′** from **F**. This is the crossing point for axial rays of light, called the nodal point, and designated **h**. At a distance of **f** from **F**, and **f′** from **F′**, there is another point on the principal axis. This is called the principal point, and is designated **p**. Its position, when the lens is regarded as having no thickness, is that of the lens. Thickness maintains **F** and **F′** as single points; but separates **h**

into two points, **h** and **h′**; and **p** into two points; **p** and **p′**. These dual points are the same distance apart.

If the above lens is turned around, so that the plane surface faces the air and the convex surface is the

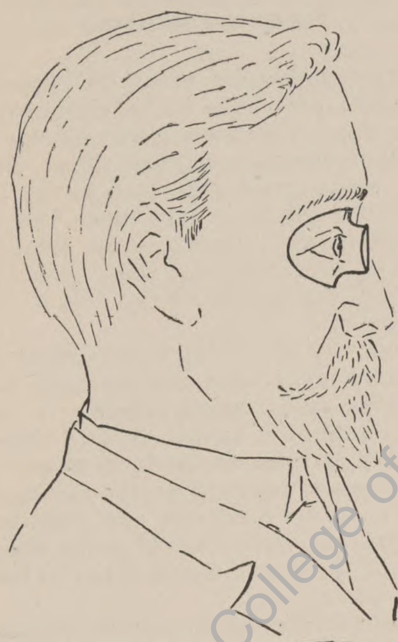

Figure 14.

submerged surface, two values will be given the system. But these values will be +6 and +7.2 D. respectively. For incident parallel rays from air the action is all at the posterior submerged surface, and is .25 of +24 = +6 D. Parallel rays from the liquid will be acted upon by the convex submerged surface

x′/n′ of $+24 = .25/1.25 = +4.8$ D. But, at emergence from the plane surface this action is increased ½ of $+4.8$, making in all $+7.2$ D.

Submarine Glasses.

Under-water spectacles must be constructed according to these principles to be effective. In submerging the eyes, the cornea, bathed by water, is practically eliminated as a dioptric agent. Hence, the lenses must make good this lost power. But the lenses submerged will have very much less than their nominal value in air. Hence, very strong positive lenses would be necessary, and these would also have to be very small, cutting down the field of vision to a very small space.

The obvious way of making under-water spectacles is to protect the eyes from contact with the water, and thus provide an air space anterior to the cornea and save it as a dioptric agent. A hard rubber mask, with soft rubber margins, in the form of two eye cups, and with the lenses mounted at the forward ends of the eye cups so as to make the eye cups, lenses and mask imperviors to water, would serve this purpose. The pressure of the water would assist in holding the mountings in place.

As the front surfaces of the lenses only are in contact with water, these should be plano. If the eyes are emmetropic, both surfaces would be plane. But, any correction of the eyes could be ground upon the posterior surfaces, so as to have their full air value. The lenses in this form could be of regular size, and the wearer, upon coming out of the water, would have his correction the same in air as in water. The hard part of the problem would be the construction of the mask, as the lenses offer no difficulty whatever.

Note: Submergence of a mirror in any liquid does not affect its value, as its value has no index factor.

But submergence of a lens-mirror affects the value of the lens element in it the same as if it were separate from the mirror. Whatever its value in the liquid, as a part of the lens-mirror it has double that value, the same as the lens of any lens-mirror.

(See Quiz III, page 81)

IV.

SEPARATED LENSES.

When two thin lenses have a common principal axis, they are termed a couplet. When together (theoretically in the same place) their combined dioptric value is the sum of the two values. It is termed the nominal value of the couplet, and the symbol for it is $\mathbf{D}_s$. The focal-length of this value is $1/\mathbf{D}_s$, and its symbol is $\mathbf{f}_s$.

Separation of the two lenses does not affect their nominal values, but does affect their combined value. It may give the couplet a value greater or less than $\mathbf{D}_s$, according to the character of the individual lenses forming the couplet. This increased or reduced value, due to separation, is the true dioptric value of the couplet when the lenses are separated. Its symbol is **D**, and its focal-length is $1/\mathbf{D}$ or **f**. The principal foci of the couplet have the same symbols as in a single lens, **F** and **F′**.

When the two lenses are separated, we may project the value of the first to the position of the second, and then unite the value of the second to it, or vice versa. For the value of the first we use the symbol **a**. For its value at the position of the second lens we use the symbol **a′**. For the value of the second lens we use the symbol **b**. Its value at the position of the first lens would be **b′**. The first or anterior lens is the one first to receive light, or nearest the source of light.

When the value of the first lens, **a**, is projected to the second, **b**, and becomes there **a′**, and is united in value with **b,** we call this combined value the "vertex" power or value of the couplet. The symbol for this

value is $\mathbf{D}_v$, and the focal-length of that value is $1/\mathbf{D}_v, = \mathbf{f}_v$. We thus have three different values for a lens couplet, each with a corresponding focal-length, besides the individual values of the lenses composing the couplet. We also have the value of each lens projected to the position of the other lens.

There remains still, as an element in the value of the couplet, the space separating the two lenses, or the distance between the two lenses of the couplet. As other values are expressed in diopters, or metric units, this space should also be in meters. The sym-

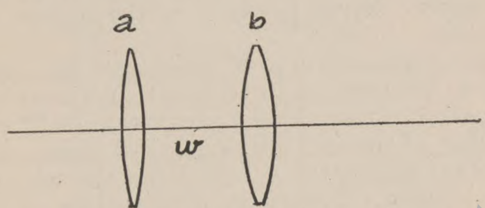

Figure 15.

bol for the space is **w,** which may represent any metric distance between the lenses of a couplet. The lenses are of course assumed to be in air.

Illustration.

If our two thin lenses are a +5 and a +8, their $\mathbf{D}_s$ value is +13, and $\mathbf{f}_s$ is 1/13 of a meter. If they are separated by a space of 3″ or .075 meters, the nominal value, $\mathbf{D}_s$, is unchanged, for that is their value when together. But their true value, **D,** becomes something different. We wish to find what this value is, also the $\mathbf{D}_v$ value, and the positions of **F′** and **F.** We also wish to know from what points a focal-length of the couplet is to be measured. As there will be two

of these points, each a focal-length from **F′** and **F,** we designate them **h′** and **h.**

If the two lenses, +5 and +8, are separated a distance of 8″ (we understand this to be metric inches, each inch being 1/40 of a meter, and therefore .984 of a regular inch) the second lens will be at the posterior principal focus of the first lens. Parallel rays of light incident at the +5 are focalized at this point. Hence, the value of the +5, projected to this point, is of infinite power. Consequently, neither the +8, nor any other lens, located at that point, can have any influence or value. It is eliminated as a value from the couplet. Hence the value of the couplet is but the +5 alone.

Since a separation of 8″ eliminates the value of the +8 from the value of the couplet; and it may be found in the same manner that a separation of 5″ eliminates the value of the +5, leaving it +8 alone, each 1″ of separation eliminates +1 D. from the nominal value of the couplet. Therefore, a separation of 3″, which is the separation given in the example, eliminates +3 from the nominal value, and the value of the couplet, with the lenses separated 3″, is +13 — 3 = +10 D. This is the true dioptric value of the couplet given, and its focal-length is 1/10 meter, or 4″ = 10 cm.

For any couplet of lenses, separated by a space, there is a unit distance of separation that affects the value of the couplet 1 D. This unit space is the focal-length of the first lens divided by the dioptric value of the second lens. If the second lens, in the above example, had been a +10, separation of 8″ would have reduced the value of the couplet from +15 to +5, or taken out the value +10. As the focal-length of +5 is 20 cm. the unit space would have been 1/10 of 20 cm. = 2 cm. But the focal-length of the first lens, divided by the dioptric value of the second is the product of their focal-lengths. Hence, the unit space

for any two lenses of a couplet is the product of their focal-lengths.

To ascertain how much the value of the couplet has been affected by the separation of the lenses, we divide the space of separation by the unit distance, or multiply it by the reciprocal of the unit space. By subtracting this value from the nominal value of the couplet we obtain its true value. Hence in the above couplet,

$$\mathbf{D} = +5 +8 -(5 \times 8 \times 3/40) = +10 \text{ D}.$$

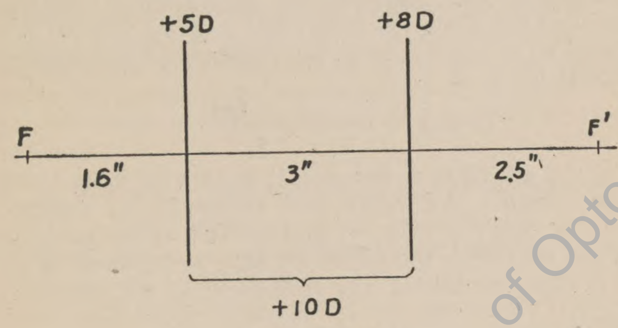

Figure 16.

Algebraic Analysis.

Using the symbols for value given heretofore, this reduces to the formula,

$$(1)\quad \mathbf{D = a + b - abw.} \text{ Hence,}$$

$$(2)\quad \mathbf{f} = 1/\mathbf{D} = \frac{1}{\mathbf{a + b - abw}}$$

These formulas may hereafter, in this section, be referred to by number. Either of them applies to any

couplet of lenses. It must be observed, however, that if any of the values are minus, the sign must be taken as in all algebraic formulas.

To obtain the other values, whose symbols have already been given, a complete analysis of the action of the couplet is necessary. We set about this by tracing the course of incident parallel rays of light to and through the couplet, taking account of each contributing element or factor as we come to it. The first of these elements is the anterior lens, whose dioptric value is symbolized by **a.** Its focal-length is then 1/**a,** and if it is a positive lens, its posterior principal focus is in the direction of **b.** In traveling toward it a space of **w,** on its arrival at **b** its distance from its own principal focus is

$$1/\mathbf{a} - \mathbf{w} = \frac{1 - \mathbf{aw}}{\mathbf{a}}$$

As the above is a distance, it cannot be added to **b** in that form. We must first reduce it to dioptric value, the dioptric value of **a** projected to the position of **b,** and therefore the value we have symbolized as **a′.** This is done by taking the reciprocal of the distance. Hence,

$$(3)\quad \mathbf{a'} = \frac{\mathbf{a}}{1 - \mathbf{aw}}$$

To this value we may now add the value **b** itself, which gives us the combined value of the two at the position of the second or final lens. The value is

$$\mathbf{a'} + \mathbf{b} = \frac{\mathbf{a}}{1 - \mathbf{aw}} + \mathbf{b}.$$

$$\text{“} \quad = \frac{\mathbf{a} + \mathbf{b} - \mathbf{abw}}{1 - \mathbf{aw}} = \frac{\mathbf{D}}{1 - \mathbf{aw}}.$$

But observe, the first member of the last equation,

since it is the combined value of **a** at **b** and **b**, is the vertex power, or the $\mathbf{D_v}$ of the couplet; and its reciprocal is the $\mathbf{f_v}$ for the same. Hence,

$$(4)\ \mathbf{D_v} = \frac{\mathbf{D}}{1 - \mathbf{aw}}$$

$$(5)\ \mathbf{f_v} = \frac{\mathbf{D}}{1 - \mathbf{aw}}$$ Tr

Since the focal-length of the couplet is **1/D,** while the vertex focal-length is **aw/D** less than that amount, it is evident that the vertex focal-length is but a part or segment of the true focal-length. Moreover, if the principal focus is posterior to **b,** this segment is also posterior to **b.** It is obvious then that the remaining segment of the true focal-length must lie anterior to **b.** Its anterior extremity, **h′** is therefore located as follows:

(6) **h′** is **aw/D** anterior to the posterior lens, **b.**

(7) $\mathbf{F'}$ is $\dfrac{1 - \mathbf{aw}}{\mathbf{D}}$ posterior to the posterior lens, **b.**

It will have to be borne in mind that the terms "anterior" and "posterior" are used in a relative sense only, and that if the distance anterior turns out to be a negative distance, it is posterior. A negative posterior distance is also actually anterior.

With respect to the anterior elements, the formulas for value and for distance are the same, except that the value **b** is substituted for that of the value of **a.** Instead of a vertex value, we may designate the combined values at **a** as the "apex" value, and symbolize it by $\mathbf{D_a}$ instead of $\mathbf{D_v}$. The true value of the couplet is the same in either direction; and the true focal-length of the couplet is the same anterior as posterior.

But, as the action of a couplet in the opposite direction brings in the factor **b** in place of **a,** the positions of **F** and **h** are quite different in an anterior direction from the couplet, and therefore the values of $\mathbf{D}_a$ and $\mathbf{f}_a$ are quite different than their counterparts posteriorally, unless the two lenses forming the couplet are duplicates. Anteriorally the formulas for values and positions are as follows:

$$(8)\ \mathbf{D}_a = \frac{\mathbf{D}}{1 - \mathbf{bw}}, \text{ or } \mathbf{b'} + \mathbf{a}.$$

$$(9)\ \mathbf{f}_a = \frac{1 - \mathbf{bw}}{\mathbf{D}}$$

(10) **h** is $\mathbf{bw/D}$ posterior to the anterior lens, **a.**

$$(11)\ \mathbf{F} \text{ is } \frac{1 - \mathbf{bw}}{\mathbf{D}} \text{ anterior to the anterior lens, } \mathbf{a}.$$

The value of **b′** in the 7th formula corresponds to the value of **a′** and the formula for it is

$$(12)\ \mathbf{b'} = \frac{\mathbf{b}}{1 - \mathbf{bw}}$$

In case the value of **a′** or **b′** is known, and we wish to determine the value of **a** or **b** from it, we can, by transpositions and reductions, obtain the expression or formula for the value of **a** in terms of **a′** or or **b** in terms of **b′.** This is projecting a value away from, instead of toward, its principal focus, except in minus lens values.

As these formulas, especially that for the value of **a** in terms of **a′,** are of considerable practical importance, and are to be used in an important connection later, we will develop the formula.

By Formula (3)

$$\mathbf{a'} = \frac{\mathbf{a}}{1 - \mathbf{aw}}.$$

By clearing of fractions,

$$\mathbf{a'} - \mathbf{a'aw} = \mathbf{a}.$$

Whence, by transpositions and changing signs,

$$\mathbf{a} + \mathbf{a'aw} = \mathbf{a'} \text{ or}$$
$$\mathbf{a}\,(1 + \mathbf{a'w}) = \mathbf{a'}, \text{ and}$$

$$(13) \quad \mathbf{a} = \frac{\mathbf{a'}}{1 + \mathbf{a'w}}$$

From Formula (12) we may obtain, in the same manner,

$$(14) \quad \mathbf{b} = \frac{\mathbf{b'}}{1 + \mathbf{b'w}}$$

These four formulas, (3), (12), (13) and (14) will be referred to when the point is reached where their use is of great practical advantage.

Optical Center.

Any couplet of lenses has also an optical center. It is on the principal axis of the couplet having a relation to the positions of **h** and **h'**. It is symbolized by the letter **o.** The relationship between **o, h** and **h'** is that of negative conjugates. That is, for the anterior lens, **h** is the negative conjugate or virtual image of **o;** while **h'** has the same relationship to it for the posterior lens. However, if these points are not between the lenses, **o** becomes the virtual image of **h** or of **h'** for the respective lenses.

The position of the optical center of a couplet may be determined by either of the following formulas, as they will be found to place it in the same position:

(15) **o** is at a point **aw**/$\mathbf{D}_s$ anterior to **b**, or
(16) **o** is at a point **bw**/$\mathbf{D}_s$ posterior to **a**.

These formulas are the same as those for the positions of **h** and **h′** except that $\mathbf{D}_s$ instead of **D** is the denominator or divisor.

Practical Application.

These formulas may now be employed for the solution of any problem that pertains to lenses separated by a space. For instance, in the combination of a +5 and +8, separated by a space of 3″ = .075 in meters,

By definition,	$\mathbf{D}_s = +13.00$
By Formula (1)	$\mathbf{D} = +10.00$
By Formula (4)	$\mathbf{D}_v = +16.00$
By Formula (8)	$\mathbf{D}_a = +25.00$

Then, as to the location of cardinal points,

By Formula (6)	**h′** is 37.5 mm. anterior to **b.**
By Formula (9)	**F′** is 62.5 mm. posterior to **b.**
By Formula (10)	**h** is 60 mm. posterior to **a.**
By Formula (11)	**F** is 40 mm. anterior to **a.**
By Formula (15)	**o** is 21+ mm. anterior to **b.**

To make images with this couplet, the focal-length system is fully applicable. An object situated at ½ focal-length of **D** = 2″ anterior to **F** will have a real image 2 focal-lengths of **D** posterior to **F′** and the image will be 2 diameters of the object.

An axillary ray of light must be directed anteriorally toward **h.** Its refraction by **a** will deviate it in such manner as to cause it to pass through **o.** On

reaching the second lens the refraction of that lens will give it an emergent direction such that, projected back, it will intersect the principal axis at **h′**. The points **h, o** and **h′** in this way mark the course of axillary rays through the couplet. The course of an axillary ray may thus be laid off geometrically, and the figure of it be drawn with mechanical drawing tools.

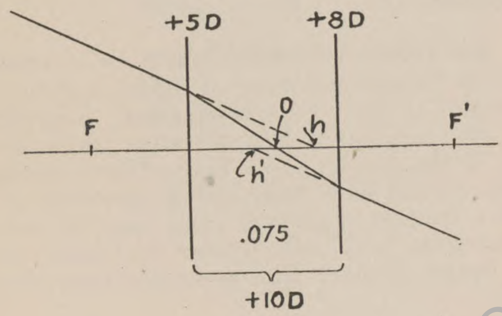

Figure 17.

Second Example.

All couplets of plus lenses follow the above, except as to elements and factors. They may give integral results or fractional ones. In the second example we will use the same lens values, but have one of the lenses minus. We will also change the space, so that our results will be, in the main, in whole numbers.

In such a couplet, with one plus and one minus member, the element **abw** is made minus by the one minus factor. But its subtraction from $\mathbf{D}_s$ converts

the whole element into a plus value. Also, in the value $\mathbf{D}_s$, their algebraic sum is really their numerical difference. The couplet is a +8 anterior lens and a —5 posterior lens, and their separation is 2″ = .05 in meters. Then, according to formulas, etc.

By definition,	$\mathbf{D}_s = +3.00$
By Formula (1)	$\mathbf{D} = +5.00$
By Formula (4)	$\mathbf{D}_v = +8.33+$
By Formula (8)	$\mathbf{D}_a = +4.00$

Any of the above values obtained by formula is greater than the nominal value of +3.00, but the value of the couplet at the vertex is greatest. The separation of a plus and minus lens therefore increases the plus value in the couplet above that of their mere sum. This gives us the idea that, by the mobility of the factors of a couplet, a greater value may be obtained than in any other way, and greater than either of the elements in the couplet. But, as to positions of cardinal points,

By Formula (6)	**h′** is 80 mm. anterior to **b.**
By Formula (9)	**F′** is 120 mm. posterior to **b.**
By Formula (10)	**h** is —50 mm. post. = ant. to **a.**
By Formula (11)	**F** is 250 mm. anterior to **a.**
By Formula (15)	**o** is 133+ mm. anterior to **b.**

It will be seen from the above that **o, h** and **h′** are all anterior to the anterior lens of the couplet. **h′** is 80 — 50 = 30 mm. forward of it; **h** is 50 mm. forward of it; and **o** is 133+ mm. forward of it. Comparing the positions of **h** and **F,** it will be found that they are 200 mm. or the focal-length of +5 D. apart. The same is true of **h′** and **F′.** Image forming by this couplet follows the rule the same as the other, the rule of focal-lengths.

An axillary ray of light to this couplet would have to be directed to **h.** As there is no lens to intercept it, it will pass through **h.** At the anterior lens, however, it will be so deviated that, projected, the line marking its course through the lens would intersect the principal axis at **o.** On reaching the second lens its course would be again deviated, and it would take a course such that, if projected, it would intersect the principal axis at **h′.**

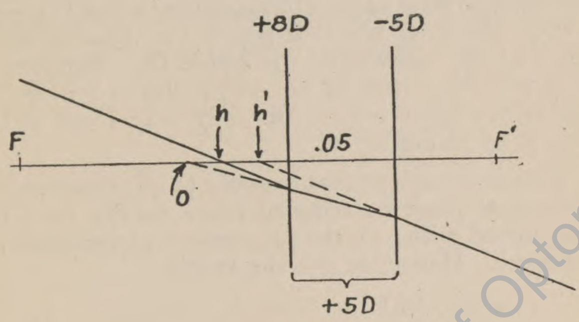

Figure 18.

Indirect Problems.

In the two examples given, the formulas were applied directly. If the unknown quantity is one of the other factors or elements, as **a, b** or **w,** it is only necessary to substitute the values of the other known quantities for the letters that symbolize them, leaving the unknown only as the symbolized value. Then, by transpositions and reductions, find the value of the unknown quantity.

In the last example, $\mathbf{a} = +8$. What is the value of $\mathbf{a}'$? By Formula (3),

$$\mathbf{a}' = \frac{\mathbf{a}}{1 - \mathbf{aw}}$$

But, since $\mathbf{a} = +8$, and $\mathbf{w} = .05$, $\mathbf{aw} = .4$; and $1 - .4 = .6$. Hence,

$$\mathbf{a}' = +8/.6 = +13\tfrac{1}{3}$$

This is the value of **a** projected to the position of **b.** But the value of **b** is —5 in the same place. Therefore their united value at that point is $+13\frac{1}{3} - 5 = +8\frac{1}{3}$, which, as you will see, is the value of the two lenses at the vertex, or the value $\mathbf{D}_v$. Suppose I do not wish this value to be $+8\frac{1}{3}$, but $+5$ only. If I employ the same lenses, the only way I can get that value is by changing **w.**

Then it is evident that in order to get this result, the value of $\mathbf{a}'$ must be reduced from $+13\frac{1}{3}$ to $+10$, so that, united with —5, the two values at that point will make $+5$. Hence, to get the result,

$$\mathbf{a}' = +10.$$

But since, by Formula (13)

$$\mathbf{a} = \frac{\mathbf{a}'}{1 + \mathbf{a}'\mathbf{w}}$$

By transpositions and reductions, we obtain,

$$(17) \quad \mathbf{w} = \frac{\mathbf{a}' - \mathbf{a}}{\mathbf{a}'\mathbf{a}}$$

Hence, by substituting the known values of **a** and $\mathbf{a}'$ in the above, we obtain,

$$\mathbf{w} = \frac{10 - 8}{80} = .025 \text{ meters}, = 25 \text{ mm.} = 1''.$$

That is, in order that a +8 and —5 lens, made up into a couplet, have a vertex power or value or +5, the lenses must be separated by a space of 25 mm. or 1 metric inch. The formula for the value of **w** is numbered Formula (17), and will be found useful for determining this space for any known or desired value. It is the difference between the nominal and vertex values of the front lens divided by their product.

Any of the symbolized values in the formulas, if unknown, may be obtained in the same manner, provided the other values in the formula are known.

(See Quiz IV, page 82)

V

FURTHER MODIFICATIONS.

Having given the methods for determining the values of lenses submerged in some medium other than air, and in different degrees of submergence; and also the methods of calculating the value of a 2-lens series or couplet, and locating the cardinal points; we may pass on to further and more complex constructions. Lenses may be combined in a 3-lens series, and a couplet or series may be more or less submerged or enclosed by media other than air.

Let me take for a first example, a 3-lens series in air. For this purpose, the purpose of making the methods of calculation clear, it will answer the purpose just as well if simple primary elements are selected, and those that we have already considered in simpler constructions. If the elements of a 3-lens series are +5, +8 and —4, and the distance from the first to the second is 3″, and from the second to the third is 2½″, the problem of getting and locating a single equivalent lens is not so difficult as it might at first appear to be.

Taking the anterior couplet, +5 and +8, separated by a space of 3″, we have already calculated its value, under Formula (1) to be +10. The location of that value, by Formula (6), places this value at a point 1½″ anterior to the +8. We may therefore consider that, in place of the +5 and +8, we have a +10, 1½″ forward of the position of the +8, or 1½ + 2½″ = 4″ anterior to the —4. As the focal-length of +10 is 4″, the —4 is located at its posterior principal focus, and therefore eliminated as a value from that couplet, which leaves it +10, and in the same place. The pos-

terior nodal point, **h′,** is at the +10 lens, and the posterior focus of the series, **F′,** is at the position of the —4 lens.

Anteriorally, however, we have a more complex calculation. The value will be the same, +10 for the 3-lens series, but the location of the points is not so simple. Our first couplet in this direction is the —4 and +8, separated by a space of 2½″ = .0625. By Formula (1).

$$\mathbf{D} = -4 + 8 - (-32 \times .0625) = -4 + 8 + 2 = +6$$

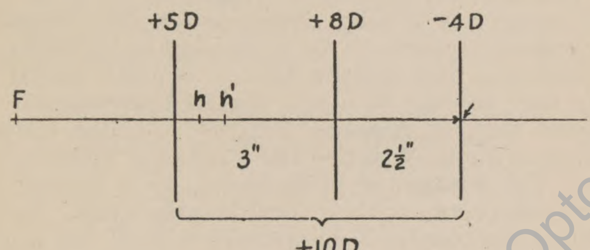

Figure 19.

By Formula (10) for this couplet, remembering that light is proceeding in the anterior direction,

h is located $-4 \times .0625 = -.25/6 = -.0416\frac{2}{3}$ posterior to +8, or anterior to it.

This carries the value toward the +5. As the +5 is .075 anterior to the +8, it is $.075 - .0416\frac{2}{3} = .033\frac{1}{3}$ anterior to the +6 value of the first couplet. We then have as the final couplet, +6 and +5 at .033⅓ distance from each other. Under Formula (1) this value reduces to +10 as before. That is,

$$\mathbf{D} = +6 + 5 - 30 \times .033\tfrac{1}{3} = +11 - 1 = +10.$$

By Formula (10) for this couplet,

$$\mathbf{h} \text{ is located at } 6 \times .033\tfrac{1}{3} = .2/10 = .02$$
posterior to $+5$.

As the 3-lens series has a value of $+10$, and therefore a focal-length of 10 cm. = .1, the principal focus, **F,** is located at a point .10 — .02 = .08 = 8 cm. anterior to the $+5$ lens.

Practical Uses.

The lens series that are made up of more than two lenses are usually employed for microscopic or telescopic purposes, or for projecting or photographic accessories. But nobody knows of what special use they may become in future optical constructions. A couplet only is required for the so-called Gallieam telescope, a plus objective and a stronger minus ocular lens. The position of these lenses, for a normal eye, is the differences of their focal-lengths apart. In that position the couplet is dioptrically neutral, but a good magnifier of distance, giving an erect view of the object.

A 2-lens series is also used practically as an erector. It is composed of two plus lenses separated by a space equal to the sum of their focal-lengths. They are dioptrically neutral. Hence a normal eye will see distant objects through the couplet normally, except that all objects will appear inverted. Used as an ocular for objective lenses that give an inverted image of a distant object, it erects the image, or makes it appear in normal position. If the lens nearest the eye is stronger than the other, it magnifies the image. If the image made by the objective lens is near the erector, the forward lens must be of sufficient power to take care of such nearness, besides focalizing correctly for the ocular lens. There is infinite variety

in the construction of these couplets, and some of them are of no value except in connection with the system of objectives to which they pertain.

The Punctumeter, though made up of but one positive lens, a +10, is constructed on the plan of a couplet; for its single lens is situated at 1 focal-length of the lens forward of the position of a correcting single lens. The other factor is a mobile target, so mounted that its distance from the lens can be changed. At a focal-length of the lens forward of it, it sends parallel rays from the target to the eye. This

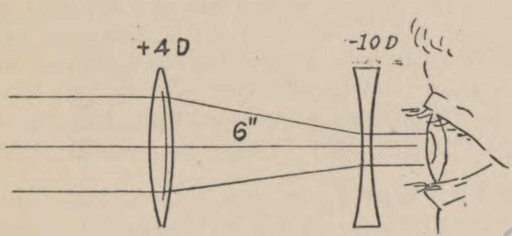

Figure 20.

gives the observer, with an emmetropic eye, normal vision of the target. But moved a centimeter in either direction, it is adapted to 1 D. of hyperopia or myopia, according to the direction it is moved.

It is said not to magnify the target. What it does is to remagnify the target that has been made to reduce by greater distance. If an object is seen at 40 rods through a 2 diameter magnifier, and is again viewed at 80 rods through a 4 diameter magnifier, it will appear of the same size under the different circumstances. That is what the Punctumeter does. Moving the target farther away reduces it, but the lens remagnifies it the same amount, so that the angle

of vision does not change. It thereby controls a lens value at the eye cup before the eye, although there is no material lens there. It is an infinitely thin lens at the eye cup, for it occupies no space whatever.

But the power of the lens value that is put before the eye by a mere movement of the target, either plus or minus, enables one to obtain, with it, a correction of any spherical error of the eye, within the ordinary range of such errors. It also provides the means of measuring the amplitude of the accommodation, including the error. It is, however, a monocular measurement, and made in the same manner as if minus lens values were placed before the eye for that purpose. It is not associated with actual nearness of an object, which calls into action the co-ordinate functions of accommodation and convergence, so that it is not to be considered as representing the full amplitude of the accommodation under natural circumstances.

The author's instrument, the Dioptrometer, is an amplified Punctumeter. It is a 3-lens system or series, and is therefore patentable and patented. In place of the target on the Punctumeter, a —10 lens is mounted, and this lens is mobile, the same as the target in the simpler instrument. This objective lens makes a virtual image of the distant object 1 focal-length in the direction of the object, or 2 focal-lengths of the +10 from it. By moving the objective —10 lens, the virtual image is moved nearer or farther from the +10. The action of the +10 focalizes the light from the virtual image, normally at 2 focals posterior to it. But this light is again intercepted, at the posterior principal focus of the +10, by a stationary —10, which parallels the rays.

The result of this combination is, that when the objective lens is in the normal position, light from the object, passing through the three lenses, emerges

from the eye lens in parallel rays, and is adapted to an emmetropic eye. Moving the objective lens moves the virtual image so as to cause the rays to converge or diverge at the ocular lens, and its action then leaves a residue of convergence of the rays or converts them into divergent rays, and for centimeters of movement, into exact dioptric values. The target may be a regular chart of test letters at 20 ft. as they will neither be magnified nor reduced, whatever position may be required to control dioptric value at the eye. There is

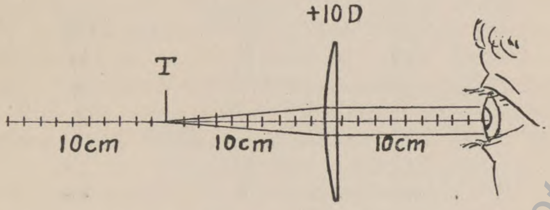

Figure 21.

otherwise the same means of measuring errors of refraction and amplitude of accommodation that the Punctumeter provides.

There is, however, this important added feature to the Dioptrometer. Its objective lens, instead of being a spherical —10, is made up of a pair of —10 cylinders, with their axles fixed at right angles to each other. Hence they may be separated, and one of them placed nearer the +10 lens than the other. This gives one meridian of the system a different power at the ocular lens than the meridian at right angles to it, and as many diopters as there are centimeters of separation between them. As the objective lens in its entirety is rotatable, the maximum and minimum power may

be located for any two meridians at right angles to each other. This provides the means of measuring astigmatism.

The non-magnifying principle of the Dioptrometer may be seen by regarding it as consisting of two opera-glasses, placed with their objectives together, and in one lens, so that one, in effect, is looking through a couplet of these glasses in reverse order. Whatever the magnification of the one, it is exactly counteracted by the other, and the distant target always

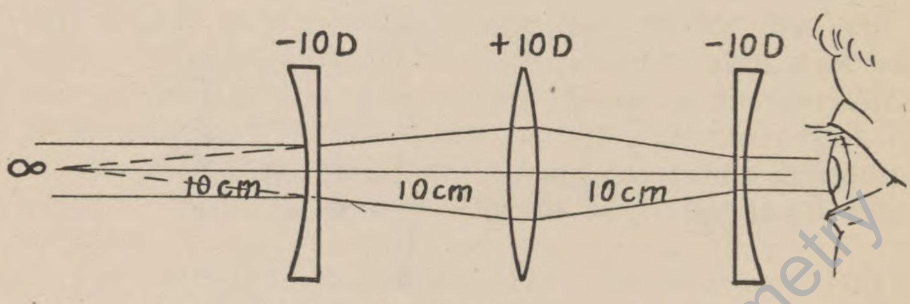

Figure 22.

remains of normal size, for the visual angle is not disturbed by any set of the objective elements. But, as such setting introduces a plus or minus value at the eye, and may be made to produce a cylindrical value in any axis, clear vision of distance would of course be affected, or the accommodation might be given more work than it can perform. There is another limitation on the instrument that I might refer to.

The instrument was first put on the market July 1st, 1914, and in quite limited numbers. Perhaps the date will recall certain facts to mind. Within a month the great world war began. Business went to the wall for a considerable period, except in war supplies. When it began to be resumed there was a woeful

scarcity of metals. Hence sales were limited to the supply first obtained, and this was soon exhausted, and no more could be made. For that reason, and that reason only, the appearance of the instrument on the market was short-lived. But, there are endings of wars as well as beginnings. This one has, however, been long drawn out. When it ends it may be possible that the Dioptrometer, improved in construction, will again make its appearance.

Prism Binoculars.

In the prism binoculars, so much employed for modern field glasses, the prisms are not used prismatically, but as total reflectors. The distant object is first focalized by a rather weak, but perfected, spherical objective lens. This gives it a long focus. The focalized light is reflected, at a short distance back of it, by a double-prism reflector, and returns in a direction parallel to its original course. It is again intercepted by a double-prism reflector, and resumes a course parallel to its first direction. This consumes considerable distance, and the light so focalized, forms a real image of the distant object near the position of the observer's eye. The ocular lens then parallels the rays before they enter the eye, and clear vision of the distant object must result. But, the ocular lens is a high magnifier, and the observer is able to see the object quite highly magnified, because of such nearness, with the assistance of the ocular lens.

In the figure the course of the light through one of the binoculars is represented, but on a flat field. Each of the double prism reflectors has an angle to the course of light of 45°, so that the two reflections of each is necessary to completely reverse its course. The slant of each mirror changes the course of the light it reflects 90°, so that two of them sends it

directly back. But the opposite reflectors stand at right angles to each other, one being at an angle of 45° and the other at 135°, so that the two full reflections of each amount to a complete horizontal and vertical reversal, or an inversion. In this respect the two double prism reflectors act as an erector of the image, and the real image that stands before the ocular lens is erect.

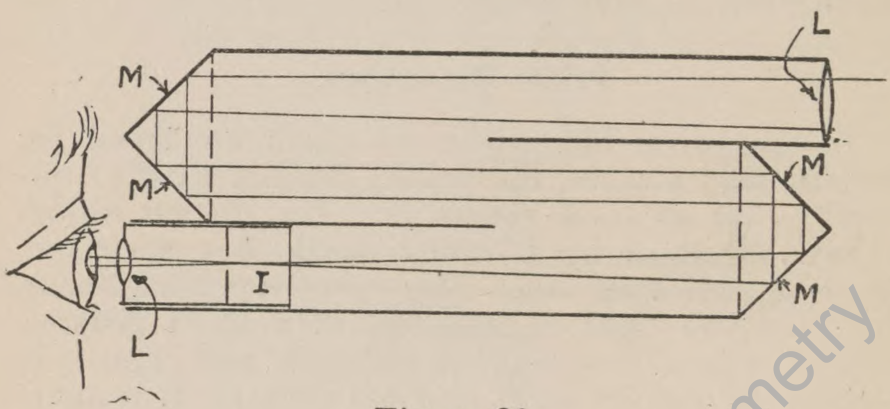

Figure 23.

The real image is of course a miniature image of the distant object. But its position is so near the eye that the strong ocular lens greatly magnifies it, and one is, as with a simple microscopic lens, looking at a small image with a high magnifier. Where the object is that gives rise to this image is a matter of "projection." If it is the picture of man and horses, or of a ship upon the water, the observer will naturally project it to the distance at which the real object would have to be to be seen as he sees it through the glasses. But, he has binocular vision of the distant object, and the objective lenses are more widely separated than the eyes, so that the stereoscopic effect is enhanced, and distance or perspective comes out even more clearly than by natural vision.

Special Devices.

The demand for trained optical service has been very much increased by the great war. Military men are not able to describe what it is they want, although they know well enough that they want it. The ingenuity of the optical man must be brought into play to foresee what is going to be required, and have it ready for that purpose when it is called for. There

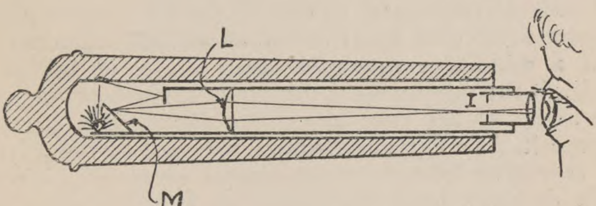

Figure 24.

is a wonderful field of invention open to those who perfect themselves in the science of optics. Out of a number of calls for special service of this kind, I will report but one. It was not for a type of instrument in common use, but for the design of one to be used for special purposes.

The bore of some of the larger rifles in the ordnace department of the government are deep and dark places, and not easy to inspect for small pits and imperfections. The means of making such inspection would naturally be of an optical character, an optical device of some kind. We suppose the bore to be 60″ deep and 2½″ in diameter, with a removable breech, or means of getting back of the bore. Looking into it from either end, when illuminated throughout, gave

too great a slant to the visual line to discover small imperfections that might be at once apparent if they could be looked directly at, especially if magnified. The demand then was for something, some optical device, that would provide these facilities. The author was merely called in to assist a local optical man on whom the call had been made.

The device presented, or the design of it, is shown in the figure. A cylindrical tube to insert in the bore, and push to the bottom. At the far end of the tube is a plane mirror, **M**, with a fixed inclination of 45° to the axis of the metal cylinder. In the center of the mirror a circular aperture of about ½″ diameter, so that a miniature nitrogen lamp below it shines up through the mirror upon the inner surface of the bore, the metal of the tube being cut away for a space to expose it. At a certain distance forward of the mirror, an objective lens, **L**, is so placed as to focalize the light received from the mirror at **I**, and make a real image of whatever is at the correct distance before it at that point, and with 4 diameters enlargement. An ocular lens to inspect the image, and the means of rotating the entire device so as to illuminate and inspect any area of the bore completed the device.

Of course there is the further practical features to the problem of volume of illumination, degree of magnification, the peculiar character of the metal surface to be inspected (a polished hollow, cylindrical, metal tube) being taken into account. These questions can only be solved by a practical trial. But, to modify the device in such manner and degree as is found necessary, is not a hard problem. The image before the ocular lens may be enlarged or reduced by a different lens in a different position. This is easily calculated, if the degree of the enlargement is stated; or it can be experimentally found by trial of different lens values.

The purpose of these illustrations is to point out to the optical student the field that is open to those who perfect themselves specially in the science of optics. Don't make it impossible, because of your lack of knowledge, for the government authorities to get any assistance from you if they need it and when they need it.

(See Quiz V, page 83)

VI.

SUBMERGED COUPLET.

Like single lenses, a couplet may be submerged, counter submerged, enclosed or semi submerged. However, anything except complete submergence gives rise to elements or factors the consideration of which will be postponed until thick lenses, and the effects of thickness upon value, has been taken up. The effects of submersion upon the individual lenses of a couplet will be that which has already been discussed. It is only with its effects upon the couplet as such that we have to deal, but that is important.

Let us take for our couplet a pair of +10 lenses, with a fixed distance of 4 cm. between them. As **ab** = 100, and **w** = .04, **abw** = .04 of 100 = 4. Therefore, by Formula (1) the couplet has an air value of +20 — 4 = +16. What it becomes upon submergence depends upon the index of the glass of which the lenses are made in connection with the index of the medium of submergence. The lenses might be of glass of a different index, but we will assume them to have the same, and that it is 1.5, so that unnecessary complexity may not be introduced into the problem. We will also assume that the medium of submergence is water, the index of which is 1.33⅓.

The relative index of the glass in the water is **n/n″** = **n′** = 1.125 for 1½/1 ⅓ = 1⅛. Hence, the density factor of value in air, **x**, is .5, and the density factor of value in the water is **x′** = .125. As the latter factor is ¼ the former, while the forms of the lenses are the same, each individual lens of the couplet has a value in water of ¼ its value in air; and ¼ of +10 is +2.5, so that each lens is +2.5 in the water.

If the couplet followed the same rule, its value in water would be ¼ of +16 = +4. Does it follow the same rule or not, and if not, why not.

The element **abw** of the couplet in air consists of the three factors. In the water, two of these factors have each been reduced to ¼ their value in air, while the third factor is unchanged. Reducing each of two factors to ¼ of their former value is a reduction of ¼ of ¼ = 1/16 in the product. Hence, while the lens

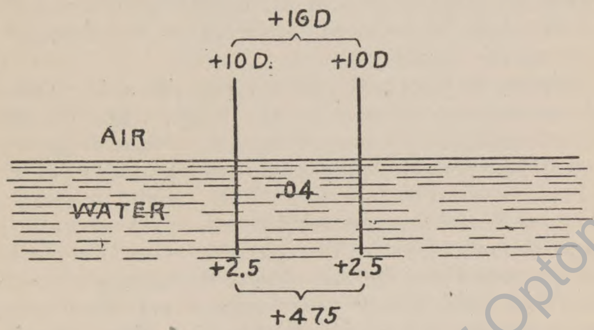

Figure 25.

values that form elements in the submerged couplet are reduced to but ¼ each, the space value that is deducted from them is reduced to 1/16, or the square of their reduction. This makes **abw** in the water have a value of but 1/16 of 4 = .25, and therefore the true value of the submerged couplet is

$$\mathbf{D} = \mathbf{a} + \mathbf{b} - \mathbf{abw} = 2.5 + 2.5 - .25 = +4.75$$

The nominal value, $\mathbf{D}_s$, of the couplet, submerged, is the sum of the individual values of the two lenses, +5. Having obtained the true value, $\mathbf{D} = +4.75$, the value of $\mathbf{D}_v$ and $\mathbf{D}_a$ may be obtained by the for-

mulas. The positions of **h** and **F**, of **h′** and **F′** and of **o** may also be determined by the regular formulas, for the couplet submerged is a pair of +2.50 lenses separated by a space of 4 cm. We may also obtain, from these values, the focal-length of the couplet, or of each of its dioptric values. Perhaps you do not see to what use this information can be put, nor indeed any of the information that is contained in this volume. But again, this particular information may be of the greatest possible value to a captain on an American submarine, and through him to the admiral, and through the admiral to the navy, and through the navy to the country.

And as to yourself, you may be the only person on the submarine, or even in the navy, who can supply the information necessary, because, in advance of the need of it, you have studied these obscure principles and made yourself ready. That sort of service is being rendered the country by those who are studying the problems of aerial navigation. The Wright boys, gliding over the hills and down the slopes near Dayton, Ohio, only a few years ago, never dreamed that the principles of aerial flight that they developed would so soon be put to the use it has been, and to the greater use it will be, and for the purposes of peace, a few years after the war is over, and the people of the earth settle down to a peaceful enjoyment of the victories won by the army and navy over a malignant and depraved ambition to subjugate the world.

In the future developments of science, the science that puts natural forces to work and relieves mankind of its burdens, the science of optics will play an essential part. It is at present far behind the other sciences, and its field is clear for the most wonderful of developments. The war has shown a shortage of optical equipments. There are many things that might

have been done, that might be done, in the war, that have not been done, and can't be, because of this shortage. The naval and military authorities do not know their own need, will not know them until they are pointed out by devices that bridge the difficulty. The time to prepare for trouble is before the trouble begins, not after it is upon us. This knowledge is not to be gotten out of the universities until someone puts it into them. Let the optically ambitious student be a pioneer in his profession, not a follower.

The empirical employment or profession of adapting lenses to the requirements of optically defective eyes is only a branch of optical science. Outside of that field, as well as in it, there is no more interesting or fascinating study. But if one tries to limit his efforts to the strictly practical and utilitarian uses of the science, success in even this limited direction is sure to escape him. It is the one who goes the limit, who allows no wall or wire entanglement to stop him on his way, that really arrives. Whether it be commercial success or not is a matter of the least consequence. Commercial success of any kind has a bitter ingredient in it; scientific success is entirely sweet and wholesome and has no bitterness in it.

Another Submersion

If the above couplet of lenses, a pair of +10's of 1.5 glass, are submerged in a transparent oil of an index of 1.25, what distance must separate them to give the couplet a **D** value of +5 in the oil? Perhaps you may say "there is no oil of so low index," and perhaps there is not; but what does that matter? We are merely "learning how" to make these calculations. We will be able to apply the principles to whatever material elements we find available. The relative index of this glass in this oil, **n/n″**, is 1.2, and there-

fore **x′** = .2, which is ⅖ or .4 of .5, the dioptric factor **x** of the glass in air. Hence the value of each lens in the oil is .4 of +10 = +4. What separation of the pair of +4 lenses will give the couplet a value of +5 in the oil? We have the values, **a**, **b**, and the desired value, **D**. What must be the value of the missing factor, **w**?

As the two individual lenses are of the same value, +4, their sum is 8 and their product is 16. Hence, in Formula (1) we have, by substituting these values for the symbols,

+5 = +8 — 16**w**, which transposes to
16**w** = 3, and **w** = 3/16 = .1875 meters = 18¾ cm.

This separation of the two lenses, in the oil, gives them a true couplet value of +5 D. The focal-length of the couplet is therefore ⅕ meter, or 20 cm. We ascertain the positions of the cardinal points as follows:

h′ is **aw/D** = .75/5 = .15 = 15 cm. anterior to **b**.

As **h′** is 15 cm. anterior to **b**, and the focal-length is 20 cm., **F′** is 20 — 15 = 5 cm. posterior to **b**. It is obvious that, in the opposite direction, the last calculation would be duplicated. Hence, **h** is 15 cm. posterior to **a**, and **F** is 5 cm. anterior to **a**.

Figure 26 represents the couplet submerged in the oil, with all of the points, including **o**, located on the principal axis, as **o** is midway between the lenses. We may calculate the action of this couplet on light from an object at any distance the same as we would calculate it for a simple spherical lens in air. For instance, if the object, **OH**, is situated at a distance of 25 cm. from the anterior lens, it is one focal of the couplet anterior to **F**. Consequently its real image will be at one focal posterior to **F′**, and the image will be of the same size as object. An axial ray of light

from **O** will take a course directly toward **h**. At the anterior lens it will be deviated toward **o** and pass through it. At the second lens it will be again deviated, taking a direction parallel with its original direction, and in alignment with **h'**.

The real image at **IK** will be of the same size as the object, **OH**, as object and image, each a focal-length of the couplet from the respective principal foci, or two focal-lengths of the couplet from the

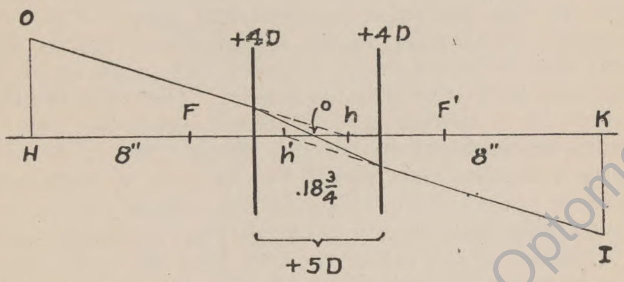

Figure 26.

respective nodal points, are in the planes of the symmetrical points. Any ray of light that passes through the anterior symmetrical plane, and on through the single lens or couplet of which it is the symmetrical plane, will pass through a point in the posterior symmetrical plane that is the same distance from the principal axis, but on the opposite side of it. By this means rays of light may be traced through a lens or couplet. Any ray, not axial to the lens or couplet, but which passes through it, has also a prescribed artificial course that takes it to a point

in the posterior symmetric plane corresponding to the point in the anterior symmetrical plane it passes through.

This course is as follows. From the point in the anterior symmetrical plane crossed by the ray, draw a straight line to the plane of the anterior nodal point. This represents the initial direction of the ray. From this point in the plane of the anterior nodal point, draw a straight line to the plane of the second nodal point, parallel with the principal axis of the lens. Then, from this point, draw a straight line through a point in the posterior symmetrical plane that corresponds, in position, to the initial point—that is, the point that is at an equal distance but in an opposite direction from the principal axis. The ray, **OMNI**, in Fig. 26, is such a ray. It is not axial, but has its course marked out in results, though not in details, for it is obvious that it will be deviated by each lens when it passes them. The actual course of the ray between the lenses, is a straight line from the point of incidence of the ray at the first lens, **m**, to the point of emergence from the second one, **m′**. As it is not an axial ray, it is out of alignment with the cardinal points, **h**, **o** and **h′**.

Other examples of submerged couplets might be given, varied by having one of the lenses of greater power than the other; one of different glass than the other; changing the fixed distance between the lenses, having one of the lenses positive and the other negative. But these, by ascertaining the relative index of the glass in the liquid, would all simplify to a couplet of lenses of fixed powers, in the liquid, at fixed distances, so that there would be nothing new in them. We cannot go into the other varieties until the effect of lens thickness has been considered, and the method of taking actual thickness of the lens into account has been determined.

Submarine Port-hole.

An under-water vessel of any kind containing human beings must provide means of admitting light from the sea into the vessel, and giving those inside of it a chance to see the water and what it contains outside of the vessel. But such a port-hole would of course have to be water tight, and glass, being impervious to water, serves both purposes. A flat plate of glass would serve the purpose very well, but it

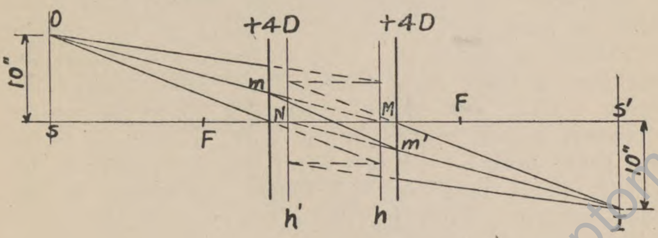

Figure 27.

might be regarded as too weak to resist the pressure of the sea. If that led to convexing the glass on the outside, the result would be a lens with one surface submerged. As a lens, if plano-convex, it would have considerable plus power, and perhaps blur the vision of an observer from the inside.

The optical problem is that of surfacing the glass in such manner as to make it, in its semi-submerged position, neutral as a lens. With outside surface convexed, a plane inner surface would support the outer plus curve, so that the inner surface must be concave. It is a somewhat elusive problem, even with the index

factors all given. Assuming that the index of the glass is 1.5, and of the water 1.33⅓, which gives a relative index of 1.125 for the glass to the water, if the lens is such as to cause parallel rays from the air within to emerge into the water as parallel rays, parallel rays from without will enter the vessel as parallel rays also, or the lens will be neutral for distant objects in either direction.

Assuming the inner surface to have a metric curvature of —3, parallel rays from the air will be acted upon for $\mathbf{x}/\mathbf{n} = \frac{1}{3}$ of $-3 = -1$ D. This will give them a divergence or metric curvature of +1 in the glass. There is left only the convex surface for them to pass, and its action is $\mathbf{x}'/1$ of the metric curvature, or $.125 = \frac{1}{8}$. The metric curvature at the emergent surface must therefore be +8 to make the action +1 D. But, there is already +1 in the metric curvature of the waves in the lens. Hence, in order that there may be +8 left after this curvature is deducted, there must be +9 metric curvature in the glass surface. That is, the convexity of the outer surface must be to the concavity of the inner surface as 9 to 3, or 3 times as great.

Whether the curvature is expressed in metrocurves or dioptric values, based on the same index, doesn't matter. A regular +3 D. outer surface and a —1 D. inner surface to the lens would satisfy all requisites. If the port-hole window were to be quite small, say 2″ or 3″ in diameter, higher powers in the same ratio might be selected, as +6 D. outer and —2 D. inner surfaces. An opening of that diameter would offer sufficient opportunity for observation from within the vessel, if there were also means of illuminating the water and objects in it so as to provide sufficient light for the purpose. This would have to be provided by a separate port-hole for that purpose, a light on the search-light order being used for the purpose.

Water, especially sea water, might be found insufficiently transparent to allow observation for any great distance, but the port-hole and illumination would together give a fairly clear view of the water and objects in it for a limited distance. A telescopic tube might even have space for use in connection with the port-hole. It would not however overcome the opacity of the water, but merely enlarge objects

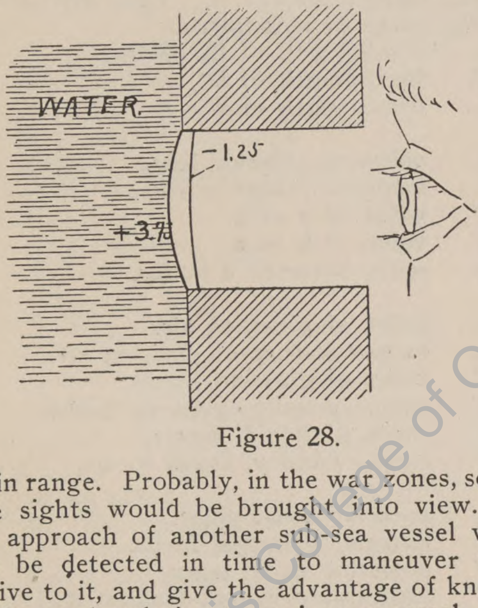

Figure 28.

within range. Probably, in the war zones, some grewsome sights would be brought into view. But the near approach of another sub-sea vessel would perhaps be detected in time to maneuver the vessel relative to it, and give the advantage of knowing just how it stands relative to one's own vessel.

If the glass of the port-hole window were quite thick, a new calculation would best be made, taking into account such thickness. This is a matter to be fully dealt with in part 3 of this series.

(See Quiz VI, page 84)

Summary of Algebraic Expressions:

Symbols:

$\mathbf{D}_s$, nominal lens value.
$\mathbf{D}$, true lens value.
$\mathbf{D}_v$, lens value at vertex.
$\mathbf{D}_a$, lens value at apex.
$\mathbf{D}'$, secondary lens value.
$\mathbf{D}_m$, mirror value.
$\mathbf{D}_{lm}$, value of lens-mirror.
$\mathbf{f}$, focal-length of $\mathbf{D}$. or $\mathbf{D}_m$.
$\mathbf{f}'$, focal-length of $\mathbf{D}'$.
$\mathbf{f}_v$, focal-length of $\mathbf{D}_v$.

$\mathbf{a}$, value of anterior lens.
$\mathbf{b}$, value of posterior lens.
$\mathbf{a}'$, value of $\mathbf{a}$ at $\mathbf{b}$.
$\mathbf{b}'$, value of $\mathbf{b}$ at $\mathbf{a}$.
$\mathbf{w}$, space between 2 lenses.

$\mathbf{n}$, index of glass, to air.
$\mathbf{x}$, excess density of glass in air.
$\mathbf{n}'$, index of glass to liquid.
$\mathbf{x}'$, excess density, glass in liquid.
$\mathbf{n}''$, index of liquid, to air.
$\mathbf{x}''$, excess density, liquid to air.

$\mathbf{h}$, anterior nodal point.
$\mathbf{F}$, anterior principal focus.
$\mathbf{h}'$, posterior nodal point.
$\mathbf{F}'$, posterior principal focus.
$\mathbf{o}$, optical center.

Formulas:

I. 1. $\mathbf{D}/\mathbf{x} = \mathbf{C}$.
2. $\mathbf{D}/\mathbf{n}$ = action curved incident surface.
3. $\mathbf{D}\mathbf{x}/\mathbf{n}$ = action emergent plane surface.

II. 1. $\mathbf{D_m} = 2\mathbf{b}/\mathbf{x}$.
2. $\mathbf{D_{1m}} = 2\mathbf{D} + 2\mathbf{b}/\mathbf{x}$.

III. 1. $\mathbf{n} = \mathbf{n'n''}$.
2. $\mathbf{n'} = \mathbf{n}/\mathbf{n''}$.
3. $\mathbf{n''} = \mathbf{n}/\mathbf{n'}$.
4. $\mathbf{x} = \mathbf{n} - 1$.
5. $\mathbf{x'} = \mathbf{n'} - 1$.
6. $\mathbf{x''} = \mathbf{n''} - 1$.
7. $$\mathbf{D'} = \frac{\mathbf{D}}{\mathbf{x}}\left(\frac{\mathbf{n}}{\mathbf{n''}} - 1\right).$$
8. $$\mathbf{D'} = \mathbf{D}/\mathbf{x}\left(\frac{\mathbf{x} - \mathbf{x''}}{\mathbf{n''}}\right).$$

IV. 1. $\mathbf{D} : \mathbf{D'} :: \mathbf{x} : \mathbf{x'}$.

V. 1. $\mathbf{D} = \mathbf{a} + \mathbf{b} - \mathbf{abw}$.
2. $\mathbf{f} = 1/\mathbf{D}$.
3. $$\mathbf{a'} = \frac{\mathbf{a}}{1 - \mathbf{aw}}.$$
4. $$\mathbf{D_v} = \frac{\mathbf{D}}{1 - \mathbf{aw}} = \mathbf{a'} + \mathbf{b}.$$
5. $$\mathbf{f_v} = \frac{1 - \mathbf{aw}}{\mathbf{D}}$$
6. $\mathbf{h'} = \mathbf{aw}/\mathbf{D}$ anterior to $\mathbf{b}$.
7. $$\mathbf{F'} = \frac{1 - \mathbf{aw}}{\mathbf{D}}$$ posterior to $\mathbf{b}$.

8. $\mathbf{D_a} = \dfrac{\mathbf{D}}{1 - \mathbf{bw}} = \mathbf{b'} + \mathbf{a}.$

9. $\mathbf{f_a} = \dfrac{1 - \mathbf{bw}}{\mathbf{D}}.$

10. $\mathbf{h} = \mathbf{bw/D}$ posterior to $\mathbf{a}$.

11. $\mathbf{F} = \dfrac{1 - \mathbf{bw}}{\mathbf{D}}$ anterior to $\mathbf{a}$.

12. $\mathbf{b'} = \dfrac{\mathbf{b}}{1 - \mathbf{bw}}.$

13. $\mathbf{a} = \dfrac{\mathbf{a'}}{1 + \mathbf{a'w}}.$

14. $\mathbf{b} = \dfrac{\mathbf{b'}}{1 + \mathbf{b'w}}.$

15. $\mathbf{o} = \mathbf{aw/D_s}$ anterior to $\mathbf{b}$.

16. $\mathbf{o} = \mathbf{bw/D_a}$ posterior to $\mathbf{a}$.

17. $\mathbf{w} = \dfrac{\mathbf{a'} - \mathbf{a}}{\mathbf{a'\,a}}.$

QUIZ BOOK
No. 3

I.

1. A lens of 1.5 glass has an anterior surface of +7c and a posterior surface of +5c. What is the action of each surface on light from an object 20″ forward of the lens?
2. If, with the above lens, the object is moved to a position 8″ forward of the lens, what then will be the action of each surface, and where will the emergent light focus?
3. A lens of 1.5 glass, having a +5 D. anterior surface, receives light from an object 20″ forward of the lens; what is the action of the surface, and the metric curvature of waves in the lens?
4. If the posterior surface of above lens is —2 D. what action will take place as the light passes out of the lens through it, and at what distance will the image be formed?
5. If a lens of standard glass (index 1.52) has an anterior surface of +6 D., what will be its action on incident parallel rays, and what will be the curvature of waves in the lens?
6. If the posterior surface of the last lens is —5 D., what will be its action on the rays of light as they emerge from the lens, and where will the emergent rays focus?
7. If a plano-convex lens of 1.52 glass has a +4 D. anterior surface, what will be its action on parallel incident rays, and what will be the action of the posterior plane surface?
8. If a bi-convex lens of 1.52 glass has a +2 D.

anterior surface, and a +4 D. posterior surface, what will be the action of each surface on light from an object 20″ from the lens?

9. If a lens of any index has +8 D. at its anterior surface and —5 D. at its posterior surface, at what distance from the lens must an object stand to make each surface act for its exact value?

10. In the last action, where will the image be formed; will it be virtual or real; and what will be its size compared with the object in the position stated?

II.

1. If you have glass of an index of 1.5, with surfacing tool to correspond, and grind a +2 D. plano-convex lens, silvering the plane surface, what power has it as a lens-mirror?

2. With the above lens-mirror, at what distance would the object have to be placed in order to produce a real image of it on a screen 5 diameters of the object?

3. What dioptric value would be given to the lens-mirror formed by silvering the convex surface of the above lens; and where would you place the object to make a real image 2 diameters of object.

4. If the object is placed 10″ forward of the last lens-mirror, will the image formed be real or virtual; where will it be; and what will be its enlargement or reduction?

5. If, with the above glass and tools, a —2.50 D. pcc. lens is ground, and the convex surface is silvered, what will be the dioptric value of the lens-mirror formed?

6. If a regular pcx. lens of the same power (+2.50)

is ground with the same glass and tools, what would be the effect of silvering the concave surface?

7. If you take a stock lens of —0.50 D. meniscus on +6 base, index of glass 1.52, and silver the convex surface, what is the dioptric power of the lens-mirror?
8. With 1.62 special glass, but with tools for 1.52 glass, I grind a plano-convex lens of +1 D. as measured by lens measure, and silver the plane surface. What is the value of $\mathbf{D}_{lm}$?
9. With glass whose index is 1.65, you grind a regular pcx. lens with 1.52 tools, so that it measures +1.00 D. with lens measure, and silver convex surface. What is $\mathbf{D}_{lm}$ value?
10. With glass of 1.52 and tools to correspond, you are asked to supply a customer with a concave mirror of 10″ focus. How near can you come to supplying the demand?

III.

1. If glass of an index of 1.5 is submerged in a liquid of 1.3, what is the relative index of the glass in such liquid?
2. If glass of an index of 1.6 is submerged in a liquid of 1.2, what is the relative index of the glass in the liquid?
3. If a bubble of air is imprisoned in glass of an index of 1.58, what is the index of the air in the glass?
4. If a +8 D. lens, of glass whose index is 1.5, is submerged in a fluid whose index is 1.3, what is its dioptric value in the fluid?
5. If a +10 D. lens of glass of 1.6 index is submerged in a fluid having an index of 1.25, what is its value in the fluid?

6. If a lens of +12 D. in air becomes +5 D. by submersion in a fluid, and the index of the glass is 1.5, what is the index of the fluid?
7. If a lens of +12 D. in air, submerged in a fluid of 1.3, becomes +7 D. in the fluid, what is the index of the glass?
8. If an air lens, imprisoned in glass of 1.5, is —3 D. in the glass, what is the metric curvature of the air lens?
9. What is the dioptric value of a +10 D. lens of 1.5 glass when it is enclosed by gelatine of an index of 1.4?
10. If a lens of 1.5 glass of +20 D. power, curved surface to the air, has water back of it, what are its dioptric values?

IV.

1. If two thin lenses, a +5 and —8, are made up into a couplet, the space between them being 5″, what is the true dioptric value of the couplet?
2. For the above couplet, what are the positions of **h, h′, F, F′,** and **o,** measured from the posterior lens?
3. Calculate or determine by formula the value of the anterior lens at the position of the posterior lens.
4. What is the vertex refraction or power of the above couplet? and how can this power be increased 1 D?
5. With a +10 and —4 thin lenses made up as a couplet, what separation of the two would give them a true refraction of +7 D.?
6. If the two lenses above are separated 2″ what is their true dioptric value as a couplet?
7. For the last couplet, determine the positions of all of the cardinal points named in Q. 2.

8. With the same lenses, +10 and —4, I desire a couplet whose vertex refraction, $\mathbf{D}_v$, is +12. What separation of the lenses is necessary?
9. Determine the $\mathbf{D}_v$ and the $\mathbf{D}_a$ value for the couplet described in Question 6.
10. With a +10 and —8, separated by a space of 9″, what is the true dioptric value of the couplet?
11. In the above couplet, where would the object have to be placed to parallel all rays passing through the couplet?
12. Where also would the object require to stand to make a real image of it 2 diameters of the object itself?

V

1. In the 3-lens series first described in text, what is the $\mathbf{D}_v$ and the $\mathbf{D}_a$ values of the series?
2. If there is a real image at a position 16″ posterior to the —4 lens of the series, where does the object stand relative to the +5 lens?
3. For the above imaging of an object by the 3-lens series, what is the size of the object if the image is 4″ high?
4. If a 3-lens series is composed of a +8, 2″ space, —5, .8″ space, and a final —8 lens, what is the value of **D** for the series?
5. For the last series, locate **h′** and **F′**; also locate **h** and **F.** What is the $\mathbf{D}_v$ and the $\mathbf{D}_a$ value of the series?
6. If you take from your trial case a —8 sph. as an ocular lens, and a +4 is held out in front of it, at what distance will the latter afford good vision of distant objects?
7. In viewing objects 20 rods away through the above couplet, how large or near do they appear to be? Which is it, larger or nearer?